

Blind Adaptive Detection Using Differential CMA for CDMA Systems

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SUMMARY

In this paper, we consider the application of adaptive linear filters adjusted by stochastic gradient algorithms to the suppression of multiple-access interference in DS/CDMA communications. It is pointed out that the linearly constrained constant modulus algorithm (LCCMA) cannot converge to the optimum point if the desired user magnitude is less than a critical value. Next, we propose to use the linearly constrained differential constant modulus algorithm (LCDCMA), and show that the LCDCMA always converges to the optimum point regardless of the desired user magnitude. Several simulation results show the following three points. First, the LCDCMA provides better performance than existing algorithms because the variance of the weight vector adjusted by the LCDCMA is less than that by existing algorithms. Second, the LCDCMA is relatively insensitive to the estimation error of the desired signal vector. Third, the LCDCMA combined with a blind channel estimation algorithm is useful. © 2001 Scripta Technica, Electron Comm Jpn Pt 3, 85(1): 1–13, 2002

Key words: Linearly constrained stochastic gradient algorithm; interference suppression; blind signal processing; constant modulus criterion; channel estimation.

1. Introduction

The performance of direct-sequence/code-division multiple-access (DS/CDMA) communication systems is limited by interference from simultaneous communicating

users, that is, multiple-access interference (MAI). Suppression of the interference is an important issue to increase the capacity of CDMA systems [1]. Receivers using a linear filter with an adjustable weight vector have been investigated actively because they can suppress MAI with the information of only the desired user even in an unknown environment [2]. Among techniques to determine the weight vector of such a linear receiver, blind techniques have received considerable attention since they need no training sequences which waste channel bandwidth [3]. Blind techniques can be classified into two categories: batch processing methods [4, 5] and adaptive methods [6, 7]. Our focus in this paper is on stochastic gradient algorithms in the latter category because of their simplicity.

The first seminal approach of blind adaptive MAI suppression in Refs. 6 and 7 has been based on the principle of the linearly constrained minimum variance (LCMV) [8] which was originally developed for adaptive array antennas. The principle of the LCMV approach is to minimize the receiver output variance without canceling the desired component. When a stochastic gradient algorithm based on the LCMV criterion (LCMVA) is used, the mean weight vector converges to an equivalent to the minimum-mean-square-error (MMSE) solution. However, the weight vector adjusted by the LCMVA fluctuates around the optimum point [7], so that error probability performance degrades. Another disadvantage of the LCMVA is that it may cancel the desired component at the receiver output if there are inaccuracies in the estimate of the desired signal vector which depends on the desired user's spreading code and timing, and is needed for linear constraints [7].

The constant modulus algorithm (CMA) is known as an alternative blind stochastic gradient algorithm for adap-

tive antenna arrays [9]. If the CMA is applied to MAI suppression, it can converge to an undesired local minimum [10]. To overcome this disadvantage, several authors have independently proposed use of the linearly constrained CMA (LCCMA) [11] for MAI suppression [12–14]. The principle of the LCCM approach is to minimize the deviation of the receiver output from a constant modulus without canceling the desired component. Consequently, the desired component can be expected to be protected from being significantly canceled even if there are inaccuracies in the estimate of the desired signal vector [13, 14]. Moreover, when the receiver output approaches the target constant modulus, the variance of the weight vector adjusted by the LCCMA can be expected to be relatively small. As for its optimality, it has been reported that if the desired user amplitude is 1 in the absence of channel noise, the LCCMA can converge to a point where the MAI is canceled completely [13]. However, no optimality analysis has as yet been made in the situation where the amplitude differs from 1. We will point out that the LCCMA cannot converge to the optimal point if the desired user magnitude is less than a critical value.

In this paper, to overcome the disadvantages of the conventional algorithms, we propose use of the linearly constrained differential CMA (LCDCMA) [15] for MAI suppression. The principle of the LCDCMA is to minimize the difference in the receiver output magnitude between two different time instants. Similar to the LCCM criterion, it can be expected that the LCDCMA is insensitive to the estimation error of the desired signal vector because it keeps the output power constant rather than minimizes it. It can also be expected that the LCDCMA provides better error probability performance because the variance of the weight vector adjusted by the LCDCMA becomes small if the output magnitude becomes constant. As for its optimality, we will show that the optimal solution can be obtained by the LCDCMA in the absence of channel noise regardless of the desired signal magnitude.

In multipath channels, the desired signal vector varies with the desired user's channel characteristics in addition to its spreading code and timing. Then, it is important for linearly constrained stochastic gradient algorithms to compensate the variation of the desired signal vector. Approaches making algorithms more insensitive to the inaccuracies in the estimate of the desired signal vector [7, 16, 17] are simple but do not necessarily provide the optimum solution. There are two approaches which can provide the optimum solution: one uses multiple constraints to extract only the desired component in each path [18], and the other estimates the correct desired signal vector [19, 20]. The former has the drawback of the high computational complexity because it needs a number of adjustable weights corresponding to each path. Therefore, we focus on the latter which is relatively simple. In Refs. 19 and 20, the

channel parameters are estimated recursively using blind algorithms and the estimated results are applied to the LCMVA. In this paper, we evaluate the performance of the receiver using the LCDCMA, instead of the LCMVA, which has desired properties as mentioned above. To estimate the desired user's channel characteristics, we employ an alternative algorithm combining two approaches proposed in Refs. 4 and 20.

2. Adaptive Receivers for CDMA Systems

2.1. Communication model

Consider asynchronous QPSK/DS/CDMA systems [1] with K users. The baseband transmitted signal from the k -th user can be represented as

$$s_k(t) = \sum_{i=0}^{\infty} A_k b_k(i) a_k(t - iT_b) \quad (1)$$

where A_k is the signal amplitude and T_b is the symbol duration. The information symbol at time i , $b_k(i) \in \{1, e^{j\pi/2}, e^{j\pi}, e^{j3\pi/2}\}$, is assumed to be an i.i.d. sequence, and the symbol streams of different users are assumed to be uncorrelated. The spreading waveforms are of the form

$$a_k(t) = \sum_{l=0}^{L_c-1} a_{kl} P_{T_c}(t - lT_c), \quad 0 \leq t < L_c T_c \quad (2)$$

where L_c is the length of the spreading sequences, $T_c (= T_b/L_c)$ is the chip duration, $a_{kl} \in \{\pm 1\}$ is the spreading sequences, and $P_{T_c}(t) = 1(0 \leq t < T_c)$, 0 (otherwise). The impulse response of the channel from the k -th user to the receiver is given by [4]

$$h_k(t) = \sum_{l=0}^{L-1} h_{kl} \delta(t - lT_c) \quad (3)$$

where L is the number of resolvable paths and assumed to be $L \ll L_c$. The coefficients $\{h_{kl}\}$ are complex channel gains. Then, the composite response, taking into account the spreading waveform and channel impulse response of the k -th user, is given by

$$g_k(t) = \int_0^t a_k(\tau) h_k(t - \tau) d\tau \quad (4)$$

Then, the received signal can be represented as

$$r(t) = \sum_{k=1}^K \sum_{i=0}^{\infty} A_k b_k(i) g_k(t - iT_b - \tau_k) + n(t) \quad (5)$$

where $\tau_k \in [0, T_b)$ is the delay time of the k -th user, $n(t)$ is a zero-mean white complex Gaussian noise process with power spectral density $N_0/2$ and assumed to be independent of the information symbols.

The structure of a receiver is shown in Fig. 1. The first user is assumed to be the desired user. The aim of the receiver is to demodulate $b_1(i)$. At the receiver, both the delay time τ_1 and spreading sequence $\{a_{1l}\}$ are assumed to be known. In the sequel, the system is assumed to be synchronized to the desired user, that is, $\tau_1 = 0$. The received signal is passed through the chip-matched filter (CMF) and sampled at every T_c .

$$r_l(i) = \frac{1}{T_c} \int_{iT_b + \tau_1 + lT_c}^{iT_b + \tau_1 + (l+1)T_c} r(t) dt \quad (6)$$

where $l = 0, 1, \dots, L_c - 1$. The CMF output samples for the i -th bit are collected in a vector $\mathbf{r}(i) = [r_0(i)r_1(i) \dots r_{L_c-1}(i)]^T$. Since the intersymbol interference and MAI are contained in $\mathbf{r}(i)$, the received vector can be written as

$$\mathbf{r}(i) = \sum_{k=1}^K A_k \left\{ b_k(i) \hat{\mathbf{h}}_k + b_k(i-1) \tilde{\mathbf{h}}_k + b_k(i-2) \bar{\mathbf{h}}_k \right\} + \mathbf{n}(i) \quad (7)$$

where $\mathbf{n}(i) = [n_0(i)n_1(i) \dots n_{L_c-1}(i)]^T$ and its l -th component is

$$n_l(i) = \int_{iT_b + \tau_1 + lT_c}^{iT_b + \tau_1 + (l+1)T_c} n(t) dt$$

The vector $\hat{\mathbf{h}}_k$ in Eq. (7) is given by [3, 2]

$$\hat{\mathbf{h}}_k = \sum_{m=0}^{L-1} h_{km} \left\{ \frac{\delta_k}{T_c} \hat{\mathbf{a}}_k^{(m+\nu_k+1)} + \left(1 - \frac{\delta_k}{T_c}\right) \hat{\mathbf{a}}_k^{(m+\nu_k)} \right\} \quad (8)$$

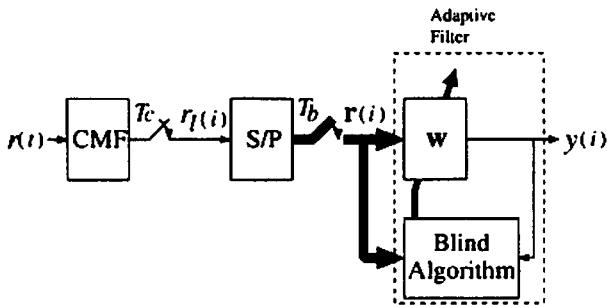


Fig. 1. Structure of receiver.

where $\nu_k = \lfloor \frac{\tau_k}{T_c} \rfloor$ ($\lfloor x \rfloor$ is the maximum integer which does not exceed x), $\delta_k = \tau_k - \nu_k T_c$, and

$$\hat{\mathbf{a}}_k^{(n)} = [0 \dots 0 a_{k0} a_{k1} \dots a_{kL_c-n-1}]^T$$

To rearrange the expression in Eq. (8), we introduce the following vector:

$$\hat{\mathbf{c}}_k^{(m)} = \frac{\delta_k}{T_c} \hat{\mathbf{a}}_k^{(m+\nu_k+1)} + \left(1 - \frac{\delta_k}{T_c}\right) \hat{\mathbf{a}}_k^{(m+\nu_k)}$$

Then, $\hat{\mathbf{h}}_k$ can be rewritten as

$$\hat{\mathbf{h}}_k = \sum_{m=0}^{L-1} h_{km} \hat{\mathbf{c}}_k^{(m)}$$

Similarly, $\tilde{\mathbf{h}}$ and $\bar{\mathbf{h}}_k$ can be written as

$$\tilde{\mathbf{h}}_k = \sum_{m=0}^{L-1} h_{km} \tilde{\mathbf{c}}_k^{(m)}$$

$$\bar{\mathbf{h}}_k = \sum_{m=0}^{L-1} h_{km} \bar{\mathbf{c}}_k^{(m)}$$

where

$$\tilde{\mathbf{c}}_k^{(m)} = \frac{\delta_k}{T_c} \hat{\mathbf{a}}_k^{(m+\nu_k+1)} + \left(1 - \frac{\delta_k}{T_c}\right) \hat{\mathbf{a}}_k^{(m+\nu_k)}$$

$$\bar{\mathbf{c}}_k^{(m)} = \frac{\delta_k}{T_c} \bar{\mathbf{a}}_k^{(m+\nu_k+1)} + \left(1 - \frac{\delta_k}{T_c}\right) \bar{\mathbf{a}}_k^{(m+\nu_k)}$$

$$\bar{\mathbf{a}}_k^{(n)} = [a_{kL_c-n} a_{kL_c-n+1} \dots a_{kL_c-1} 0 \dots 0]^T$$

$$\bar{\mathbf{a}}_k^{(n)} = [a_{k2L_c-n} a_{k2L_c-n+1} \dots a_{kL_c-1} 0 \dots 0]^T$$

Separating the desired component from other components in $\mathbf{r}(i)$, we then have

$$\mathbf{r}(i) = A_1 b_1(i) \hat{\mathbf{h}}_1 + \mathbf{H} \mathbf{A} \mathbf{b}(i) + \mathbf{n}(i) \quad (9)$$

where

$$\mathbf{A} = \text{diag}[A_1 A_1 A_2 A_2 A_2 \dots A_K A_K A_K]$$

$$\mathbf{b}(i) = [b_1(i-1) b_1(i-2) b_2(i) b_2(i-1) b_2(i-2) \dots b_K(i) b_K(i-1) b_K(i-2)]^T$$

$$\mathbf{H} = [\tilde{\mathbf{h}}_1 \tilde{\mathbf{h}}_1 \tilde{\mathbf{h}}_2 \tilde{\mathbf{h}}_2 \tilde{\mathbf{h}}_2 \dots \tilde{\mathbf{h}}_K \tilde{\mathbf{h}}_K \tilde{\mathbf{h}}_K]$$

We call $\hat{\mathbf{h}}_1$ in Eq. (9) the desired signal vector. To express the desired signal vector as the combination of the spreading sequence and channel impulse response, the following matrix and vector are defined:

$$\mathbf{C}_1 = \begin{bmatrix} \hat{c}_1^{(0)} & \hat{c}_1^{(1)} & \dots & \hat{c}_1^{(L-1)} \end{bmatrix} \quad (10)$$

$$\mathbf{h}_1 = [h_{10} \ h_{11} \ \dots \ h_{1L-1}]^T \quad (11)$$

Then, the desired signal vector becomes

$$\hat{\mathbf{h}}_1 = \mathbf{C}_1 \mathbf{h}_1 \quad (12)$$

2.2. Linear adaptive filtering

When the received signal vector $\mathbf{r}(i)$ is given as an input, the output of a linear filter is given by

$$y(i) = \mathbf{w}^H \mathbf{r}(i) \quad (13)$$

where \mathbf{w} is a weight vector.

The mean square error (MSE) between the training signal $\{b_1(i)\}$ and filter output is given by

$$J_l(\mathbf{w}) = \frac{1}{2} E \left[|b_1(i) - y(i)|^2 \right] \quad (14)$$

A receiver whose weight vector minimizes the MSE is known as the MMSE receiver [2]. The weight vector which minimizes $J_l(\mathbf{w})$ is known as the MMSE solution and given by $\mathbf{w}_o = A_1 \mathbf{R}^{-1} \hat{\mathbf{h}}_1$, where $\mathbf{R} = E[\mathbf{r}(i)\mathbf{r}^H(i)]$. When the channel noise is negligible, the MMSE solution can cancel the interference completely [4]. In this paper, the weight vector which can cancel the interference completely is referred to as the optimum solution. Our attention in this paper focuses on techniques to obtain the optimum solution without the use of the training sequences.

One such technique is the LCMV. The basic idea of the LCMV is to minimize the output variance

$$J_m(\mathbf{w}) = \frac{1}{2} E \left[|y(i)|^2 \right] \quad (15)$$

under a linear constraint $\mathbf{C}_1^T \mathbf{w} = \mathbf{g}$ [6–8]. Consider the case where the impulse response of the channel \mathbf{h}_1 is known and the constraint vector \mathbf{g} is chosen as

$$\mathbf{g} = \frac{\hat{\mathbf{h}}_1}{\|\hat{\mathbf{h}}_1\|} \quad (16)$$

Then, the linear constraint becomes $\mathbf{w}^H \hat{\mathbf{h}}_1 = \|\hat{\mathbf{h}}_1\|$, and the weight vector which minimizes $J_m(\mathbf{w})$ can be obtained by using the method of Lagrange multipliers as $\mathbf{w}_l = \|\hat{\mathbf{h}}_1\| \mathbf{R}^{-1} \hat{\mathbf{h}}_1 / (\hat{\mathbf{h}}_1^H \mathbf{R}^{-1} \hat{\mathbf{h}}_1)$. Since the direction of the vector is equal to that of the MMSE solution, the error probability performance of the LCMV receiver and MMSE receiver becomes identical. However, if there exists the estimation error of the desired signal vector, performance degradation is incurred due to cancellation of the desired component. It

should also be noted that when a stochastic gradient method, referred to as LCMVA, is used to minimize J_m , the performance also degrades due to the fluctuation of the weight vector. The LCMVA is given by

$$\mathbf{w}(i+1) = \mathbf{P} \{ \mathbf{w}(i) - \mu_l \mathbf{r}(i) y^*(i) \} + \mathbf{F} \quad (17)$$

where μ_m is the step gain and

$$\mathbf{P} = \mathbf{I} - \mathbf{C}_1 (\mathbf{C}_1^T \mathbf{C}_1)^{-1} \mathbf{C}_1^T \quad (18)$$

$$\mathbf{F} = \mathbf{C}_1 (\mathbf{C}_1^T \mathbf{C}_1)^{-1} \mathbf{g} \quad (19)$$

3. LCCMA Receivers

3.1. Analysis of cost function

The constant modulus (CM) criterion-based techniques [12–14] which use the knowledge that a transmitted waveform in DS/CDMA systems has a constant envelope, minimize the following cost function:

$$J_c(\mathbf{w}) = \frac{1}{4} E \left[\left| |y(i)|^2 - 1 \right|^2 \right] \quad (20)$$

under a linear constraint $\mathbf{C}_1^T \mathbf{w} = \mathbf{g}$. Since the criterion attempts to keep the output magnitude constant, it can be expected that the desired component is not canceled completely even if there is an error in the desired signal vector estimate [13, 14]. In Ref. 13, the property of stationary points of the cost function $J_c(\mathbf{w})$ has been analyzed in the case of $A_1 = 1$ and $L = 1$. In the sequel, the property of the stationary points is investigated in more general cases.

Let us consider the noiseless case. Under the constraint in Eq. (16), the filter output is given by

$$y(i) = A_1 b_1(i) \|\hat{\mathbf{h}}_1\| + \mathbf{w}^H \mathbf{H} \mathbf{A} \mathbf{b}(i) \quad (21)$$

The vector $\mathbf{u} \in C^J$ is defined as

$$\mathbf{u} = \mathbf{A} \mathbf{H}^H \mathbf{w} \quad (22)$$

Each component of this vector, u_i , can be regarded as the response to a virtual interfering user which is introduced to model the effect of asynchronous access and multipath. The number of virtual interfering users is $J = 3K - 1$. If the columns in $\mathbf{A} \mathbf{H}^H$ are linearly independent, it is sufficient to investigate the property of the stationary points of J_c which is expressed in terms of \mathbf{u} rather than \mathbf{w} [21]. We use the following gradient operator [22] to obtain the stationary points of J_c :

$$\frac{\partial}{\partial \mathbf{u}^*} = \left[\frac{\partial}{\partial u_{1R}} + j \frac{\partial}{\partial u_{1I}}, \dots, \frac{\partial}{\partial u_{JR}} + j \frac{\partial}{\partial u_{JI}} \right]^T \quad (23)$$

where the subscripts R and I denote real and imaginary part, respectively. In addition, the following Hessian [23] is used to investigate the property of the stationary points:

$$\mathbf{Q}J_c = \begin{bmatrix} \frac{\partial}{\partial \mathbf{u}} \left(\frac{\partial J_c}{\partial \mathbf{u}^*} \right)^T & \frac{\partial}{\partial \mathbf{u}} \left(\frac{\partial J_c}{\partial \mathbf{u}} \right)^T \\ \frac{\partial}{\partial \mathbf{u}^*} \left(\frac{\partial J_c}{\partial \mathbf{u}^*} \right)^T & \frac{\partial}{\partial \mathbf{u}^*} \left(\frac{\partial J_c}{\partial \mathbf{u}} \right)^T \end{bmatrix} \quad (24)$$

Taking into account the fact that the information symbol sequences are i.i.d. ones and mutually uncorrelated, the cost function can be expressed as

$$\begin{aligned} J_c(\mathbf{u}) &= \frac{1}{4} A_1^4 \|\mathbf{h}_1\|^4 - \frac{1}{2} A_1^2 \|\mathbf{h}_1\|^2 + \frac{1}{4} \\ &\quad + (A_1^2 \|\mathbf{h}_1\|^2 - \frac{1}{2}) \mathbf{u}^H \mathbf{u} + \frac{1}{2} (\mathbf{u}^H \mathbf{u})^2 \\ &\quad - \frac{1}{4} \sum_{i=1}^J |u_i|^4 \end{aligned} \quad (25)$$

The i -th component of the gradient vector is given by

$$\frac{\partial J_c}{\partial u_i^*} = (A_1^2 \|\mathbf{h}_1\|^2 - \frac{1}{2} + \sum_{j \neq i} |u_j|^2 + \frac{1}{2} |u_i|^2) u_i \quad (26)$$

Moreover, the entries of the Hessian are given by

$$e_{ii} = \frac{\partial^2 J_c}{\partial u_i \partial u_i^*} = A_1^2 \|\mathbf{h}_1\|^2 - \frac{1}{2} + \sum_{j=1}^J |u_j|^2 \quad (27)$$

$$e_{ij} = \frac{\partial^2 J_c}{\partial u_i \partial u_j^*} = u_i^* u_j, \quad j \neq i \quad (28)$$

$$s_{ii} = \frac{\partial^2 J_c}{\partial u_i^2} = \frac{1}{2} (u_i^*)^2 \quad (29)$$

$$s_{ij} = \frac{\partial^2 J_c}{\partial u_i \partial u_j} = u_i^* u_j^*, \quad j \neq i \quad (30)$$

It is clear that the right-hand side of Eq. (26) becomes zero when $u_i = 0$. In other words, $\mathbf{u} = \mathbf{0}$ is a stationary point. The point $\mathbf{u} = \mathbf{0}$ is the optimum one where all interference components become zero. Next, let us clarify whether only the point is a minimum or not.

1) The case of $A_1^2 \|\mathbf{h}_1\|^2 > 1/2$:

In this case, the parenthesis of the right-hand side in Eq. (26) is always positive. Thus, u_i must be zero so that the right-hand side in Eq. (26) becomes zero. Therefore, then, $\mathbf{u} = \mathbf{0}$ is the unique stationary point. Then, the Hessian is

$$\mathbf{Q}J_c = (A_1^2 \|\mathbf{h}_1\|^2 - \frac{1}{2}) \mathbf{I} > \mathbf{0} \quad (31)$$

and is positive definite. We can conclude that $\mathbf{u} = \mathbf{0}$ is the unique minimum.

2) The case of $A_1^2 \|\mathbf{h}_1\|^2 = 1/2$:

The right-hand side of Eq. (26) becomes $(\sum_{j \neq i} |u_j|^2 + |u_i|^2/2) u_i$. Since this term is nonzero if $u_i \neq 0$, it can be zero if and only if $u_i = 0$. Thus, $\mathbf{u} = \mathbf{0}$ is the unique stationary point. Then, the cost function is

$$J_c(\mathbf{u}) = \frac{1}{16} + \frac{1}{2} \sum_{i=1}^J \sum_{j \neq i} |u_i|^2 |u_j|^2 + \frac{1}{4} \sum_{i=1}^J |u_i|^4 \quad (32)$$

Consider a point $\bar{\mathbf{u}} \neq \mathbf{0}$ which is slightly moved from the stationary point. Then, since the change of the cost function is

$$\begin{aligned} J_c(\bar{\mathbf{u}}) - J_c(\mathbf{0}) &= \frac{1}{2} \sum_{i=1}^J \sum_{j \neq i} |\bar{u}_i|^2 |\bar{u}_j|^2 + \frac{1}{4} \sum_{i=1}^J |\bar{u}_i|^4 \\ &> 0 \end{aligned} \quad (33)$$

we can conclude that $\mathbf{u} = \mathbf{0}$ is the unique minimum.

3) The case of $A_1^2 \|\mathbf{h}_1\|^2 < 1/2$:

This case can be classified into three cases.

3-a) The case of $u_i = 0$ ($\forall i$):

As mentioned above, this is the optimum solution.

Then, since the Hessian is

$$\mathbf{Q}J_c = (A_1^2 \|\mathbf{h}_1\|^2 - \frac{1}{2}) \mathbf{I} < \mathbf{0} \quad (34)$$

that is, negative definite, $\mathbf{u} = \mathbf{0}$ is not a minimum.

3-b) The case of $u_i \neq 0$ and $u_j = 0$ ($\forall j \neq i$):

At a stationary point, dividing the right-hand side of Eq. (26) by u_i and rearranging it, we obtain $|u_i|^2 = -2A_1^2 \|\mathbf{h}_1\|^2 + 1$. Then, entries of the Hessian are

$$e_{kk} = -A_1^2 \|\mathbf{h}_1\|^2 + \frac{1}{2} \quad (35)$$

$$e_{kl} = 0, \quad l \neq k \quad (36)$$

$$s_{kk} = \begin{cases} \frac{1}{2} (u_i^*)^2 & k = i \\ 0 & k \neq i \end{cases} \quad (37)$$

$$s_{kl} = 0, \quad l \neq k \quad (38)$$

Then, since the k -th principal submatrices of the Hessian, Δ_k , are

$$\Delta_k = \begin{cases} (-A_1^2 \|\mathbf{h}_1\|^2 + \frac{1}{2})^k > 0, & 1 \leq k \leq J+i-1 \\ 0, & J+i \leq k \leq 2J \end{cases} \quad (39)$$

then $\mathbf{Q}J_c$ is semi-positive definite. Thus, the point where only one response is nonzero is a minimum.

3-c) The case of $u_i \neq 0$ ($1 \leq i \leq R$) and $u_j = 0$ ($j > R$):

In this case, $R(> 1)$ responses are nonzero. Let us assume without loss of generality that the responses from the first to R -th virtual user are nonzero. At a stationary point, dividing the right-hand side of Eq. (26) by u_i and rearranging it, we obtain $|u_i|^2 = 2A_1^2 \|\mathbf{h}_1\|^2 - 1 + 2\mathbf{u}^H \mathbf{u}$. The right-hand side of this expression for nonzero u_i is independent of i . Thus, since $\mathbf{u}^H \mathbf{u} = R|u_i|^2$, we obtain

$$|u_i|^2 = \frac{2A_1^2 \|\mathbf{h}_1\|^2 - 1}{1 - 2R} \quad (40)$$

Then, the first principal submatrix of the Hessian is

$$\Delta_1 = \frac{A_1^2 \|\mathbf{h}_1\|^2 - \frac{1}{2}}{1 - 2R} > 0 \quad (41)$$

Moreover, the second principal submatrix is

$$\Delta_2 = -3 \left(\frac{A_1^2 \|\mathbf{h}_1\|^2 - \frac{1}{2}}{1 - 2R} \right)^2 < 0 \quad (42)$$

Thus, we can conclude that since the Hessian $\mathbf{Q}J_c$ is indefinite, this point is not a minimum.

From 1) and 2), we can find that $\mathbf{u} = \mathbf{0}$ is the unique minimum if $A_1^2 \|\mathbf{h}_1\|^2 \geq 1/2$. Moreover, from 3), a necessary condition to guarantee that $\mathbf{u} = \mathbf{0}$ is a minimum is $A_1^2 \|\mathbf{h}_1\|^2 \geq 1/2$. Thus, it can be concluded that the sufficient and necessary condition to guarantee that the solution where the interference is canceled completely is the unique minimum is $A_1^2 \|\mathbf{h}_1\|^2 \geq 1/2$. If this condition is not satisfied, the LCCMA receiver cannot cancel the interference.

3.2. LCCMA

A stochastic gradient algorithm to minimize $J_c(\mathbf{w})$ is given by

$$\begin{aligned} \mathbf{w}(i+1) &= \mathbf{P} \{ \mathbf{w}(i) - \mu_c (|y(i)|^2 - 1) \mathbf{r}(i) y^*(i) \} \\ &+ \mathbf{F} \end{aligned} \quad (43)$$

where μ_c is the step gain. If the output magnitude $|y|$ is close to unity, little performance degradation due to the weight fluctuation is incurred because the update of the weight

vector is very small. However, if \mathbf{g} is given by Eq. (16), the magnitude of the response to the desired signal vector becomes $A_1 \|\mathbf{h}_1\|$ because of the linear constraint. Therefore, if $A_1 \|\mathbf{h}_1\|$ is far from unity, performance degradation can be caused by the large weight fluctuation.

4. LCDCMA Receivers

4.1. Analysis of cost function

In the area of adaptive array antennas, the use of the linear constrained differential constant modulus (LCDCM) criterion has been considered [15]. It minimizes the following cost function under a linear constraint $\mathbf{C}_1^T \mathbf{w} = \mathbf{g}$:

$$J_d(\mathbf{w}) = \frac{1}{4} E [|y(i)|^2 - |y(i-D)|^2]^2 \quad (44)$$

This criterion forces the output magnitude at i -th time to be equal to that at $(i-D)$ -th time. Thus, like the LCCMA receivers, we can expect that the desired component is not canceled completely even if there exists the estimation error of the desired signal vector. In Ref. 15, the case of $D = 1$ was considered. In the case of $D \geq 1$, we can expect that the output becomes constant modulus due to the constraint when the interference is suppressed. In the situation considered in this paper, when $D = 1$ and 2, it is difficult to analyze the property of the cost function because common signal components are contained in both $y(i)$ and $y(i-D)$. Thus, in the analysis below, we assume $D \geq 3$. In Section 6, simulation results for $D = 1$ and 3 are compared.

Next, let us consider the property of the stationary points of the cost function J_d in noiseless situation. The weight vector is assumed to be linearly constrained by Eq. (16). Taking into account the fact the information symbol sequences are i.i.d. ones and mutually uncorrelated, when $D \geq 3$, the cost function can be written as

$$J_d(\mathbf{u}) = A_1^2 \|\mathbf{h}_1\|^2 \mathbf{u}^H \mathbf{u} + \frac{1}{2} (\mathbf{u}^H \mathbf{u})^2 - \frac{1}{2} \sum_{i=1}^J |u_i|^4 \quad (45)$$

The i -th component of the gradient vector is given by

$$\begin{aligned} \frac{\partial J_d}{\partial u_i^*} &= (A_1^2 \|\mathbf{h}_1\|^2 + \mathbf{u}^H \mathbf{u} - |u_i|^2) u_i \\ &= (A_1^2 \|\mathbf{h}_1\|^2 + \sum_{j \neq i} |u_j|^2) u_i \end{aligned} \quad (46)$$

Since $A_1^2 \|\mathbf{h}_1\|^2 > 0$, the parenthesis in the right-hand side of Eq. (46) is always positive. Thus, the right-hand side of Eq. (46) becomes zero only if $u_i = 0$. Therefore, $\mathbf{u} = \mathbf{0}$ is the

unique stationary point. Next, let us evaluate the Hessian. Entries of the Hessian are given by

$$e_{kk} = A_1^2 \|\mathbf{h}_1\|^2 + \sum_{j \neq k}^J |u_j|^2 \quad (47)$$

$$e_{kl} = u_k u_l^*, \quad l \neq k \quad (48)$$

$$s_{kk} = 0 \quad (49)$$

$$s_{kl} = u_k^* u_l^*, \quad l \neq k \quad (50)$$

At the optimum stationary point $\mathbf{u} = \mathbf{0}$, the Hessian is

$$\mathbf{Q}J_d = A_1^2 \|\mathbf{h}_1\|^2 \mathbf{I} > \mathbf{0} \quad (51)$$

Thus, unlike LCCMA, the point where interference is canceled completely is the unique minimum regardless of the desired signal magnitude $A_1 \|\mathbf{h}_1\|$. Note that the solution is equivalent to the MMSE solution because the latter solution can also cancel the interference completely [4] if the channel noise is negligible.

4.2. LCDCMA

A stochastic gradient algorithm which minimizes $J_d(\mathbf{w})$ is given by

$$\begin{aligned} \mathbf{w}(i+1) &= \mathbf{P} \left[\mathbf{w}(i) - \mu_d \{ |y(i)|^2 - |y(i-D)|^2 \} \right. \\ &\quad \left. \cdot \{ \mathbf{r}(i)y^*(i) - \mathbf{r}(i-D)y^*(i-D) \} \right] + \mathbf{F} \end{aligned} \quad (52)$$

where μ_d is the step gain. We refer to it as the LCDCMA. We can expect that the weight fluctuation by the LCDCMA can be small because the term $(|y(i)|^2 - |y(i-D)|^2)$ becomes small if the output magnitude settles a constant modulus.

5. Blind Channel Estimation Algorithm

As mentioned in the previous section, the constraint vector \mathbf{g} must be set to Eq. (16) to obtain the optimum solution by the LCDCMA. This means that the impulse response of the channel \mathbf{h}_1 must be known. In this paper, we focus on the following channel estimation methods based on the subspace approach [24]:

$$\min_{\mathbf{g}} \mathbf{g}^H \mathbf{B} \mathbf{g} \quad \text{subject to } \|\mathbf{g}\| = 1 \quad (53)$$

where $\mathbf{B} = \mathbf{C}_1^T [\mathbf{I} - \mathbf{V}_s \mathbf{V}_s^H] \mathbf{C}_1$. In this paper, we employ an adaptive technique based on this subspace approach [4, 20].

If $[\mathbf{C}_1 \mathbf{H}]$ has full column rank, $\mathbf{g} = \mathbf{h}_1 / \|\mathbf{h}_1\|$ (and its scaled version) is the unique solution of (53), so that the correct channel estimation can be obtained [5]. In (53), \mathbf{V}_s is a matrix whose columns are the eigenvectors corresponding to the ξ largest eigenvalues of \mathbf{R} . ξ is the number of the vector corresponding to the signal subspace, and satisfies $\lambda_\xi > \sigma^2$, $\lambda_{\xi+1} = \sigma^2$ (σ^2 is the noise variance) when the eigenvalues $\lambda_i (i = 1, \dots, L_c)$ of \mathbf{R} are arranged in decreasing order. Since ξ depends on the conditions, such as the number of users, spreading sequences used, and so on, it cannot be known at the receiver. Then, techniques using AIC can be employed to estimate ξ [4]. Although this paper assumes that ξ is known, as will be shown in Section 6, we can expect that the performance degradation due to the estimate error of ξ is small because of the robustness of the LCDCMA unless the estimation error is large.

Subspace-based adaptive estimation techniques have been proposed in Refs. 4 and 20. In Ref. 20, a minor component analysis technique is used to obtain the eigenvector corresponding to the minimum eigenvalue of the matrix \mathbf{B} , and the number of parameters to be estimated is L . On the other hand, in Ref. 4, a penalty function method is used to obtain the eigenvector corresponding to the minimum eigenvalue of the matrix $\mathbf{D}^H \mathbf{D} (\mathbf{D} = [\mathbf{V}_s, \mathbf{C}_1])$, and the number of parameters to be estimated is $L + \xi$. It has been known that minor component analysis techniques may diverge [26]. Thus, in this paper, we use the penalty function method to obtain the eigenvector corresponding to the minimum eigenvalue of the matrix \mathbf{B} . Consequently, divergence of the algorithm could be avoided and the number of parameters to be estimated could be small. As in Ref. 4, the following function is defined:

$$q(\mathbf{g}, c) = \frac{1}{2} \mathbf{g}^H \mathbf{B} \mathbf{g} + \frac{c}{4} (\mathbf{g}^H \mathbf{g} - 1)^2 \quad (54)$$

where c is a positive constant chosen to be larger than the minimum eigenvalue of \mathbf{B} . Using the analytical results in Ref. 4, we can show that a scaled version of the minimum eigenvector of \mathbf{B} is the stable stationary point of $q(\mathbf{g}, c)$. Now, we need \mathbf{V}_s to solve the eigenvalue problem. It might be obtained by subspace tracking algorithms [4, 20]. A recursive algorithm where the signal subspace \mathbf{V}_s is computed by the PASTd algorithm [25] and $q(\mathbf{g}, c)$ is minimized by the steepest descent method simultaneously, is summarized as follows:

Step 1) Initialization

Set constants c , β , and μ_g , appropriately.

$$\mathbf{g}(0) = [1 \ 0 \ \dots \ 0]^T$$

$$\mathbf{u}_k = \mathbf{e}_k, \quad k = 0, 1, \dots, \xi - 1$$

$$\lambda_k(0) = 1, \quad k = 0, 1, \dots, \xi - 1$$

Step 2) Signal subspace estimation

$$\mathbf{x}_0(i) = \mathbf{r}(i)$$

For $k = 0 : \xi - 1$ Do

$$y_k(i) = \mathbf{u}_k^H(i-1)\mathbf{x}_k(i)$$

$$\lambda_k(i) = \beta\lambda_k(i-1) + |y_k(i)|^2$$

$$\mathbf{u}_k(i) = \mathbf{u}_k(i-1) + \{\mathbf{x}_k(i) - \mathbf{u}_k(i-1)y_k(i)\} y_k^*(i)/\lambda_k(i)$$

$$\mathbf{x}_{k+1}(i) = \mathbf{x}_k(i) - \mathbf{u}_k(i)y_k(i)$$

End

Step 3) Channel estimation

$$\mathbf{V}_s(i) = [\mathbf{u}_0(i) \ \mathbf{u}_1(i) \ \cdots \ \mathbf{u}_{\xi-1}(i)]$$

$$\mathbf{B}(i) = \mathbf{C}_1^T [\mathbf{I} - \mathbf{V}_s(i)\mathbf{V}_s^H(i)] \mathbf{C}_1$$

$$\mathbf{g}(i) = \mathbf{g}(i-1) - \mu_g \{\mathbf{B}(i)\mathbf{g}(i-1) + c\{\mathbf{g}^H(i-1)\mathbf{g}(i-1) - 1\}\mathbf{g}(i-1)\}$$

Step 4) Repeat from Step 2 to Step 3 until \mathbf{g} converges.

where \mathbf{e}_k is a vector whose k -th component is 1 and other components are 0.

6. Simulation Results

Performances of the LCDCMA have been evaluated via computer simulations. The LCDCMA was compared with the LMS algorithm, LCMVA, and LCCMA. When stochastic gradient algorithms are used, although performance degradation due to the weight fluctuation is small if a large number of data can be used for training, it is not always practical. Thus, we considered the case where the number of initial training data is relatively small.

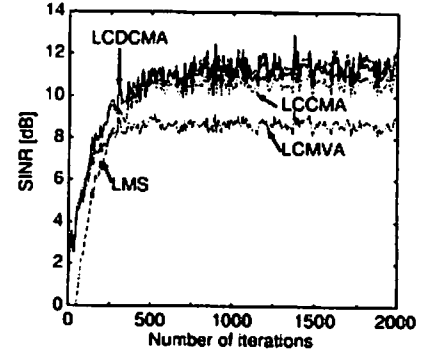
An asynchronous DS/CDMA system with $K = 5$ users was considered. The spreading sequences were Gold sequences of length 31. The number of channel paths was set to $L = 3$. The channel impulse response $\{h_{km}\}$ were chosen using complex Gaussian random generator and set to $\|\mathbf{h}_k\|^2 = 1$. Differentially encoding was employed since there is an arbitrary phase ambiguity in channel estimation results. In the sequel, the energy of the desired signal per bit is denoted by E_b , $E_b = A_1^2 T_b / 4$. All the interfering signals were assumed to have the same energy which is denoted by E_i . The energy ratio between the desired signal and interference was set to be $E_i/E_b = 10$ dB. In the following, unless otherwise mentioned, $E_b/N_0 = 13$ dB, $A_1 = 10\sqrt{2}$, $D = 3$, and the initial weights of the LCMVA, LCCMA, and LCDCMA were $\mathbf{w}(0) = \mathbf{F}$, and that of the LMS algorithm

was $\mathbf{w}(0) = \mathbf{0}$. All results were obtained by averaging over 100 different samples.

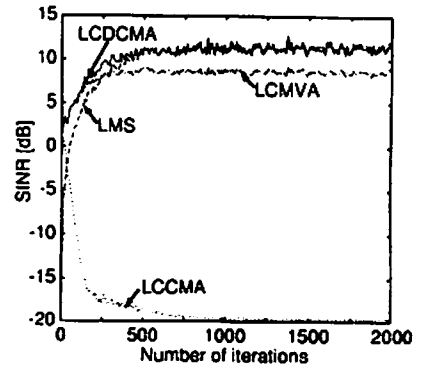
In Fig. 2, time evolution of the signal to interference and noise ratio (SINR) is shown when the desired signal amplitude is $A_1 = 10\sqrt{2}$ and $0.1\sqrt{2}$. SINR at time i was computed by

$$\text{SINR}(i) = \frac{|A_1 \mathbf{w}^H(i) \hat{\mathbf{h}}_1|^2}{\mathbf{w}^H(i) \mathbf{R}_n(i) \mathbf{w}(i)}$$

where $\mathbf{R}_n(i)$ is the autocorrelation matrix of the sum of interference and noise components and was computed by $\mathbf{R}_n(i) = \sum_{k=1}^{100} \mathbf{r}^{(k)}(i) \mathbf{r}^{(k)H}(i) / 100$, where the superscript (k) represents the k -th sample, and $\mathbf{r}^{(k)}(i) = \mathbf{r}^{(k)}(i) - A_1 b_1^{(k)}(i) \hat{\mathbf{h}}_1$. The step size of the LCDCMA was chosen so that SINR converges in approximately 500 iterations. As a result, when $A_1 = 10\sqrt{2}$, $\mu_d = 2 \times 10^{-9}$, and when $A_1 = 0.1\sqrt{2}$, $\mu_d = 2 \times 10^{-1}$. The step sizes of the LMS algorithm, LCMVA, and LCCMA were chosen so that the convergence rate of SINR is almost the same as that by the LCDCMA. As a result, when $A_1 = 10\sqrt{2}$, $\mu_l = 10^{-6}$, $\mu_m = 10^{-6}$, $\mu_r = 2 \times 10^{-9}$, and when $A_1 = 0.1\sqrt{2}$, $\mu_l = 10^{-2}$, $\mu_m = 10^{-2}$, $\mu_r = 10^{-2}$. In



(a) $A_1 = 10\sqrt{2}$



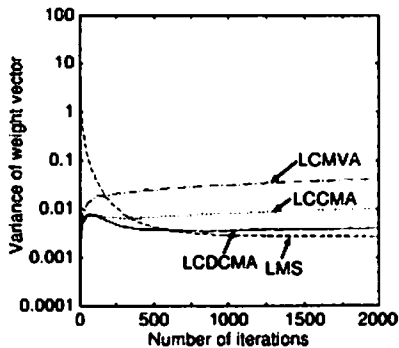
(b) $A_1 = 0.1\sqrt{2}$

Fig. 2. Performance comparison for stochastic gradient algorithms.

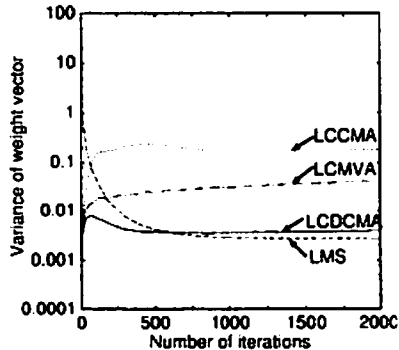
Fig. 3, the variance of the weight vectors is shown. Figure 4 shows that the magnitude of the response to each virtual user at filter output, $u_k = (\mathbf{A}\mathbf{H}^H)_k \mathbf{w}$, for the LCDCMA and LCCMA $[(\mathbf{A}\mathbf{H}^H)_k]$ represents the k -th column vector in $\mathbf{A}\mathbf{H}^H$. Figure 5 shows bit error probabilities obtained by averaging the decision error rate for 10,000-bit data after 1000 training iterations.

First, let us consider the LCMVA. We can find that the SINR of the LCMVA is lower than that of the LMS algorithm regardless of A_1 . As mentioned in Section 2.2, this may be due to the weight vector fluctuation. It is seen from Fig. 3 that the weight variance of the LCMVA is larger than that of the LMS algorithm. Although the weight variance can be smaller if a small step gain is used, then the convergence rate becomes slow. Due to the weight fluctuation, we can observe from Fig. 5 that its error probability is worse than that of the LMS algorithm.

Next, we consider the LCCMA. It is seen from Figs. 4(b) and 4(d) that all the interference can be suppressed when $A_1 = 10\sqrt{2}$, but the 6th virtual user signal cannot be suppressed when $A_1 = 0.1\sqrt{2}$. We can observe from Fig. 2(b) that the SINR of the LCCMA is very low when $A_1 = 0.1\sqrt{2}$. Moreover, it can be observed from Fig. 5(b) that its error probability is near 0.5 regardless of E_b/N_0 .

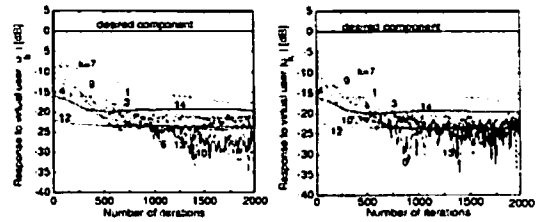


(a) $A_1 = 10\sqrt{2}$



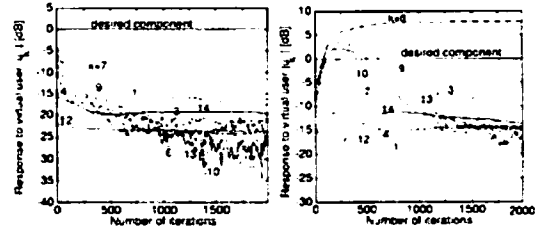
(b) $A_1 = 0.1\sqrt{2}$

Fig. 3. Variance of weight vector.



(a) LCDCMA ($A_1 = 10\sqrt{2}$)

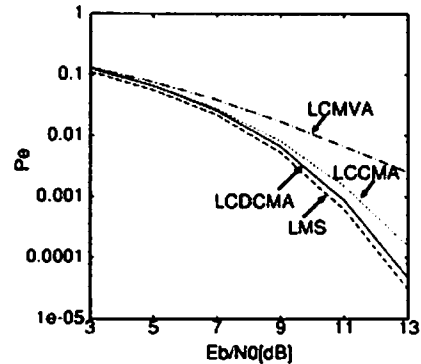
(b) LCCMA ($A_1 = 10\sqrt{2}$)



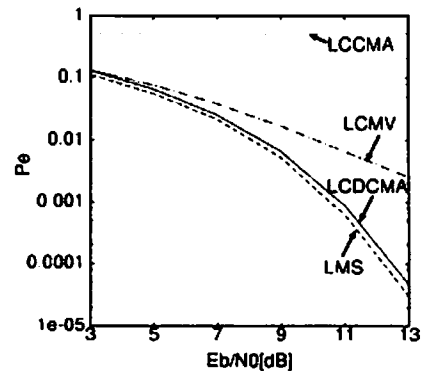
(c) LCDCMA ($A_1 = 0.1\sqrt{2}$)

(d) LCCMA ($A_1 = 0.1\sqrt{2}$)

Fig. 4. Response to virtual user.



(a) $A_1 = 10\sqrt{2}$



(b) $A_1 = 0.1\sqrt{2}$

Fig. 5. Bit error probability performance.

According to the analytical result in Section 3.1, the LCCMA cannot suppress the interference in the case of $A_1 < 0.5\sqrt{2}$. Note that the analysis is based on statistical average and does not take into account the channel noise. On the other hand, the simulation is based on stochastic approximation by the LCCMA and takes into account the channel noise. Although there exists such a difference between the analysis and simulation, we could find in simulation that interference suppression ability degrades as A_1 becomes small if A_1 is smaller than $0.5\sqrt{2}$. On the other hand, when $A_1 = 10\sqrt{2}$, we can observe from Figs. 2(a) and 5(a) that achievable SINR by the LCCMA is lower than those of the LCDCMA and LMS algorithm, and its error probability is worse than those of the other algorithms. As mentioned in Section 3.2, the reason may be that the weight variance becomes large since the output magnitude becomes $A_1 \|h_1\|$ due to the linear constraint when the interference is suppressed. In practice, we can see from Fig. 3(a) that the weight variance of the LCCMA is larger than those of the LCDCMA and LMS algorithm.

Last, the LCDCMA is considered. One can see from Figs. 4(a) and 4(c) that the interference can be suppressed regardless of the desired signal amplitude as shown in the analysis of Section 4.1, even in the presence of channel noise. From Figs. 2 and 5, we can observe that the SINR and error probability characteristics of the LCDCMA are almost the same as those of the LMS algorithm regardless of the desired signal amplitude. Moreover, as expected, we can confirm from Fig. 3 that the weight variance of the LCDCMA is relatively small and almost the same as that of the LMS algorithm. In the above results, the parameter D was set to 3. Although we chose $D \geq 3$ for the purpose of analysis in Section 4.1, D is desired to be small so as to save the memory requirement. In Fig. 6, the error probabilities for $D = 1$ and 3 are shown. The two curves overlap. We can expect from the result that D can be set to 1 in practice. The

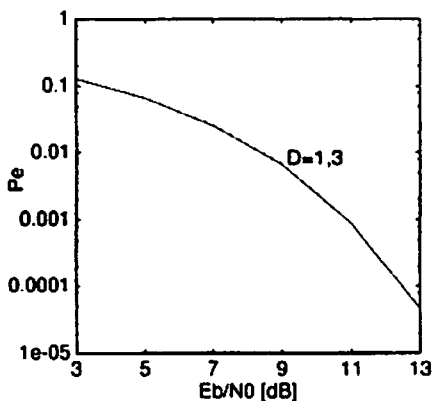


Fig. 6. Comparison of bit error probabilities for LCDCMA with $D = 1$ and 3.

analysis in the case of $D < 3$ is an important issue for future investigation.

Next, the robustness of the LCDCMA against the estimation error of the desired signal vector is considered. In the case where there is an error $\delta h_{1l} = h_{1l} - h'_{1l}$ in the estimate h'_{1l} of a channel coefficient h_{1l} , the error probability against the standard deviation of δh_{1l} is shown in Fig. 7. The estimation error δh_{1l} was chosen by Gaussian random generator with zero mean. The step gains were determined in the same manner as the previous simulations. We can see from Fig. 7 that the LCDCMA is superior to both the LCMVA and LCCMA. One may claim that the robustness against estimation error is not clear from Fig. 7 since the LCDCMA is superior to the others even if there is no estimation error. Since the robustness against the estimation error depends on the cost function, it is difficult to evaluate it from the results by stochastic gradient algorithms due to the weight fluctuation. Instead of the stochastic gradient algorithms, we evaluate it by using the steepest descent methods. The error probability obtained by the steepest descent methods using statistics obtained from 1000 samples is shown in Fig. 8. The standard deviation of the estimation error was set to 0.1. As expected, the LCDCM criterion outperforms the LCMV criterion.

Now, we consider the case where the interference suppression is carried out by using the estimated channel characteristics. In this case, since there may exist error in channel estimates, performance degradation in interference suppression may be incurred. As shown in the previous results, the LCDCM-based method is robust against the estimation error of the desired signal vector, so that performance degradation can be expected to be small. First, we consider the channel estimation algorithm described in Section 5. In order to obtain the number of vectors which represent the signal subspace, the eigenvalues of the auto-

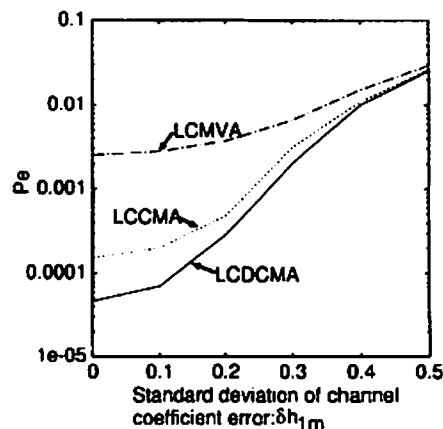


Fig. 7. Bit error probability of stochastic gradient algorithms in the presence of channel estimation error.

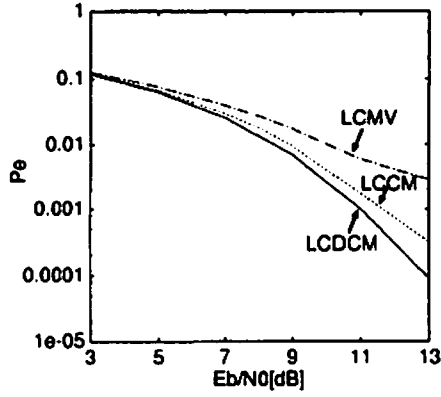


Fig. 8. Bit error probability of steepest descent methods in the presence of channel estimation error.

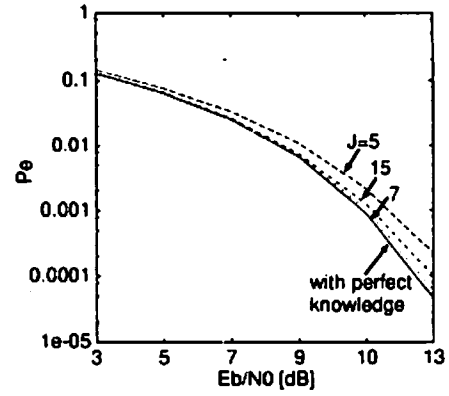


Fig. 11. Bit error probability by LCDCMA based on estimated channel impulse response.

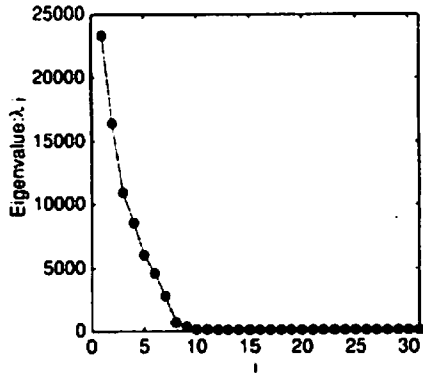


Fig. 9. Eigenvalue of autocorrelation matrix of received signal.

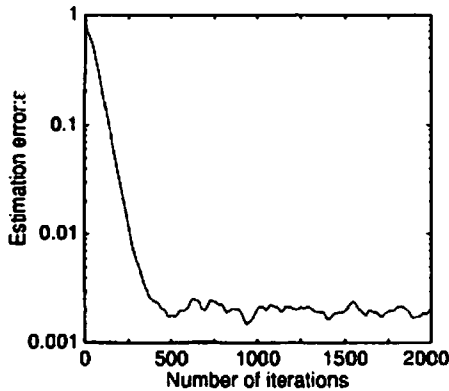


Fig. 10. Learning curve of channel estimation.

correlation matrix of the received signal are examined. In Fig. 9, we plot the eigenvalues in decreasing order. We can find from the result that the seven largest eigenvalues are dominant. In the case where the estimate of ξ is set to $\hat{\xi} = 7$, Fig. 10 shows the time evolution of the following estimation error:

$$\epsilon(i) = \left\| \frac{\mathbf{g}(i)}{\|\mathbf{g}(i)\|} e^{-j \arg(g_1(i))} - \frac{\mathbf{h}_1}{\|\mathbf{h}_1\|} e^{-j \arg(h_{11})} \right\|^2$$

Estimation accuracy which guarantees that the performance degradation is small was determined from Fig. 7. The parameters were chosen so as to achieve accuracy in approximately 250 iterations. As a result, $c = 10$, $\beta = 0.95$, and $\mu_g = 5 \times 10^{-4}$. It can be seen from Fig. 10 that the error decreases with time.

Lastly, the error probability of the LCDCMA with the linear constraint using the channel impulse response estimated by the channel estimation algorithm is shown in Fig. 11. In addition to the case where the estimate of ξ is set to be $\hat{\xi} = 7$, the results for $\hat{\xi} = 5$ and 15 are also shown as the cases containing the estimation error of ξ . Moreover, the result of the perfect channel estimate shown in Fig. 5 is also shown as a reference. In the case of $\hat{\xi} = 7$, the result is almost the same as that of the perfect estimation case. Moreover, in the cases of $\hat{\xi} = 5$ and 15, the degradation of error probability is very small. Therefore, we can conclude that the LCDCMA with the channel estimation algorithm is effective.

7. Conclusions

In this paper, we have considered stochastic gradient algorithms for adaptive receivers to suppress the MAI in

DS/CDMA communications. It has been shown that the LCCMA cannot suppress the interference when the desired signal magnitude is less than a critical value. On the other hand, we have shown that the LCDCMA can suppress the interference regardless of the desired signal magnitude. It has also been shown that the weight variance of the LCDCMA is small, so that the LCDCMA can provide superior error probability performance. It has been demonstrated that the LCDCMA is more robust to the estimation error of the desired signal vector compared to the LCMVA. Moreover, we have shown the feasibility of the LCDCMA with the blind channel estimation algorithm.

Although this paper has considered Eq. (20) as a CM criterion, alternative methods have been considered in Refs. 11 and 14: the desired modulus of the output is adjusted adaptively, or is set to be an appropriate constant instead of 1. Since the convergence properties of these methods have not been cleared, we have not considered these methods. Theoretical consideration of these methods and comparison with the LCDCMA are interesting issues and might be studied in the future.

Further researches include finding conditions to ensure the stability of the algorithm and clarifying the relation between the convergence rate and weight fluctuation.

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