

# Joint Blind Compensation of Inter-Block Interference and Frequency-Dependent IQ Imbalance

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**SUMMARY** In this letter, we propose a blind adaptive algorithm for joint compensation of inter-block interference (IBI) and frequency-dependent IQ imbalance using a single time-domain equalizer. We combine the MERRY algorithm for IBI suppression with the differential constant modulus algorithm to compensate for IQ imbalance. The effectiveness of the proposed algorithm is shown through computer simulations.

**key words:** *blind algorithm, inter-block interference, IQ imbalance, time-domain equalization*

## 1. Introduction

Orthogonal frequency division multiplexing (OFDM) is a promising technique for high data rate transmission in highly time dispersive channels. A drawback of OFDM is residual inter-block interference (IBI) which occurs when the length of the channel impulse response is longer than that of cyclic prefix (CP) inserted between data blocks. To compensate for IBI, blind channel shortening methods using a linear time-domain equalizer (TEQ) are known to be effective because they require no training symbols and provide high spectral efficiency. A well-known method is the MERRY (Multicarrier Equalization by Restoration of Redundancy) algorithm [1].

Meanwhile, in a direct-conversion architecture, IQ imbalance which occurs when the matching between the I and Q branches is imperfect also affects OFDM performance severely. Specifically, in wideband systems, IQ imbalance becomes frequency-dependent due to the impairments in analog components such as low-pass filters [2], [3]. There are two approaches for blind compensation of frequency-dependent IQ imbalance in the absence of IBI [2]–[4]. In frequency-domain approaches [3], [4], compensation is performed for each subcarrier, and this results in high computational cost when the number of subcarriers is large. A time-domain approach using a TEQ proposed in [2] is preferable to the frequency-domain ones in terms of complexity but exhibits limited performance due to a large fluctuation of a weight vector in an adaptive process.

Recently, it has been reported in [5] that a TEQ adjusted by MERRY can suppress IBI completely even in the

presence of frequency-dependent IQ imbalance but cannot compensate for the IQ imbalance. Thus, an immediate idea to compensate for both IBI and IQ imbalance is to employ a two-stage approach where IBI is suppressed by the first TEQ using MERRY and IQ imbalance is compensated for by the second TEQ. However, the use of the second TEQ causes a recurrence of undesired IBI.

In this letter, we consider joint compensation of both IBI and frequency-dependent IQ imbalance by a single TEQ, which not only simplifies a receiver structure but also avoids the IBI recurrence. While we use MERRY for the IBI suppression, for the IQ imbalance compensation, we employ the differential constant modulus algorithm (DCMA) [6] which has a small weight vector fluctuation and thus improves IQ imbalance suppression performance.

## 2. Joint IBI and IQ Imbalance Compensation

### 2.1 Problem Formulation

We consider an OFDM transmission system with  $N$  subcarriers and  $D$  receive antennas. The  $n$ th time-domain OFDM block of length  $N$  is defined as  $[x_{nN} \cdots x_{nN+N-1}]^T = \mathbf{F}^H \mathbf{s}_n$ , where  $\mathbf{F}$  is the DFT matrix,  $\mathbf{s}_n = [s_{nN} \cdots s_{nN+N-1}]^T$  consists of  $N$  QPSK symbols with the variance  $\sigma_s^2$ , and the superscripts  $T$  and  $H$  are the transpose and the conjugate transpose of a vector or matrix, respectively. Then, a CP of length  $P$  is appended to each block to form the  $n$ th transmission block of length  $Q = N + P$  as  $[u_{nQ} \cdots u_{nQ+Q-1}] = [x_{nN+N-P} \cdots x_{nN+N-1} x_{nN} \cdots x_{nN+N-1}]$ .

Combined frequency-independent and frequency-dependent IQ imbalance of the  $d$ th antenna can be modeled by two filters with impulse responses of length  $L_r$  [2], [5]:

$$\begin{aligned} b_{1,l}^{(d)} &= (f_{1,l}^{(d)} + \gamma^{(d)} f_{Q,l}^{(d)} e^{-j\phi^{(d)}})/2, \quad d = 0, \dots, D-1, \\ b_{2,l}^{(d)} &= (f_{1,l}^{(d)} - \gamma^{(d)} f_{Q,l}^{(d)} e^{j\phi^{(d)}})/2, \quad l = 0, \dots, L_r - 1 \end{aligned} \quad (1)$$

where  $\gamma^{(d)}$  and  $\phi^{(d)}$  are the frequency-independent gain mismatch and phase mismatch, and  $f_{1,l}^{(d)}$  and  $f_{Q,l}^{(d)}$  are mismatched filters in I and Q branches. We define the effective channels  $a_{i,j}^{(d)}$  as the convolution of a physical channel  $h_j^{(d)}$  of order  $L_h$  and  $b_{i,l}^{(d)}$ :

$$a_{1,j}^{(d)} = \sum_{l=0}^{L_r-1} b_{1,l}^{(d)} h_{j-l}^{(d)}, \quad a_{2,j}^{(d)} = \sum_{l=0}^{L_r-1} b_{2,l}^{(d)} h_{j-l}^{(d)*} \quad (2)$$

for  $j = 0, \dots, L_h + L_r - 1$ , where the superscript  $*$  represents complex conjugate. Then, the input vector to a TEQ

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of length  $L_t$  can be written as

$$\tilde{\mathbf{r}}_k = \left[ \mathbf{r}_k^T \cdots \mathbf{r}_{k-L_t+1}^T \right]^T = \mathbf{A}_1 \tilde{\mathbf{u}}_k + \mathbf{A}_2 \tilde{\mathbf{u}}_k^* + \tilde{\mathbf{n}}_k \in \mathbb{C}^{L_t D \times 1} \quad (3)$$

where  $\mathbf{r}_k \in \mathbb{C}^{D \times 1}$  denotes the received signal vector whose  $d$ th element  $r_k^{(d)}$  is the received signal of the  $d$ th antenna at time  $k$ , an effective channel impulse response matrix  $\mathbf{A}_i \in \mathbb{C}^{L_t D \times L_c}$  is a block Toeplitz matrix with the first row block  $[\mathbf{a}_{i,0} \cdots \mathbf{a}_{i,L_h+L_r-1} \mathbf{0} \cdots \mathbf{0}]$  where  $\mathbf{a}_{i,j} = [a_{i,j}^{(0)} \cdots a_{i,j}^{(D-1)}]^T$ ,  $\tilde{\mathbf{u}}_k = [u_k \cdots u_{k-L_c+1}]^T$ ,  $\tilde{\mathbf{n}}_k$  is a noise vector, and  $L_c = L_t + L_r + L_h - 1$ .

We consider to jointly suppress the effects of both IBI and IQ imbalance by a single TEQ which has a weight vector  $\mathbf{g}$  which can be divided into two parts as  $\mathbf{g} = [\tilde{\mathbf{g}}_1^T \tilde{\mathbf{g}}_2^T]^T$  with  $\tilde{\mathbf{g}}_i = [g_{i,0} \cdots g_{i,L_t D-1}]^T$ . Then, the output of the TEQ becomes

$$y_k = \mathbf{g}^H \hat{\mathbf{r}}_k = \tilde{\mathbf{g}}_1^H \tilde{\mathbf{r}}_k + \tilde{\mathbf{g}}_2^H \tilde{\mathbf{r}}_k^* = \mathbf{c}_1^H \tilde{\mathbf{u}}_k + \mathbf{c}_2^H \tilde{\mathbf{u}}_k^* + n_k \quad (4)$$

where  $\hat{\mathbf{r}}_k = [\tilde{\mathbf{r}}_k^T \tilde{\mathbf{r}}_k^H]^T$  and  $n_k = \tilde{\mathbf{g}}_1^H \tilde{\mathbf{n}}_k + \tilde{\mathbf{g}}_2^H \tilde{\mathbf{n}}_k^*$ . The total channels consisting of a physical channel, an IQ imbalance filter and a TEQ can be represented by  $\mathbf{c}_1 = [c_{1,0} \cdots c_{1,L_c-1}]^T = \mathbf{A}_1^H \tilde{\mathbf{g}}_1 + \mathbf{A}_2^T \tilde{\mathbf{g}}_2$  corresponding to a direct component and  $\mathbf{c}_2 = [c_{2,0} \cdots c_{2,L_c-1}]^T = \mathbf{A}_2^H \tilde{\mathbf{g}}_1 + \mathbf{A}_1^T \tilde{\mathbf{g}}_2$  corresponding to a mirror component. After the CP removal, the  $n$ th TEQ output vector can be written as

$$\mathbf{y}_n = \begin{bmatrix} y_{nQ+P} \\ \vdots \\ y_{nQ+Q-1} \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{r}}_{nQ+P}^T \\ \vdots \\ \hat{\mathbf{r}}_{nQ+Q-1}^T \end{bmatrix} \mathbf{g}^* = \hat{\mathbf{R}}_n \mathbf{g}^*. \quad (5)$$

Then, the  $n$ th DFT output can be written as

$$\mathbf{z}_n = \begin{bmatrix} z_n[0] \\ \vdots \\ z_n[N-1] \end{bmatrix} = \mathbf{F} \mathbf{y}_n = \mathbf{F} \hat{\mathbf{R}}_n \mathbf{g}^* = \mathbf{V}_n \mathbf{g}^*. \quad (6)$$

## 2.2 Proposed Algorithm

The purpose of TEQ is to make  $\mathbf{c}_1$  zero except the first  $P+1$  elements to cancel IBI and to make  $\mathbf{c}_2$  zero to cancel IQ imbalance by appropriately determining the vector  $\mathbf{g}$ . To determine  $\mathbf{g}$ , we propose the following cost function:

$$J(\mathbf{g}) = \mathbb{E} \left[ |y_{nQ+P-1} - y_{nQ+Q-1}|^2 \right] + \xi \sum_{i=0}^{N-1} \mathbb{E} \left[ \left| |z_n[i]|^2 - |z_{n-1}[i]|^2 \right|^2 \right] \quad (7)$$

where  $\xi$  is a parameter to balance the two terms. The norm constraint  $\|\mathbf{g}\| = 1$  is imposed to avoid  $\mathbf{g} = \mathbf{0}$ . The first term on the right-hand side is the MERRY cost function [1], which can suppress IBI, i.e.,  $c_{1,i} = 0$ ,  $i = P+1, \dots, L_c-1$ , by restoring the CP property at the TEQ output [5]. The second term is introduced to suppress the effect of IQ imbalance by borrowing the idea of DCMA [6]. It tries to make the magnitudes of successive DFT outputs equal. Suppose that IBI is

suppressed and channel noise is negligible. Then, the DFT outputs  $z_n[i]$  contains the IQ imbalance component in addition to the desired component which is  $H_i s_{nN+i}$  where  $H_i$  is the frequency gain of the total channel  $\mathbf{c}_1$  at the  $i$ th subcarrier. Clearly, if the IQ imbalance component is suppressed, the second term in (7) becomes zero because both  $|z_n[i]|^2$  and  $|z_{n-1}[i]|^2$  become  $\sigma_s^2 |H_i|^2$ . From the above discussion, the cost function  $J$  can be minimized when both IBI and IQ imbalance are suppressed. Conversely, it is interesting to clarify that both IBI and IQ imbalance can be suppressed when the cost function is minimized. It is, however, cumbersome to show it theoretically because of the non-convexity of the cost function. Instead, we verify the proposed method through simulations in the next section.

To minimize the cost function, we employ a stochastic gradient algorithm. The TEQ vector  $\mathbf{g}_n$  is updated every time one OFDM block is received. The proposed algorithm is summarized as follows:

- 1) Initialization:  $n = 0$ ,  $\mathbf{g}_0 = [1 \ 0 \ \cdots \ 0]^T$ ,  $z_0[i] = 0$ ,  $\forall i$ .
- 2) Obtain  $\mathbf{V}_n = \mathbf{F} \hat{\mathbf{R}}_n$ ,  $\mathbf{z}_n = \mathbf{V}_n \mathbf{g}_n$ , and  $\mathbf{z}_{n-1} = \mathbf{V}_{n-1} \mathbf{g}_{n-1}$ .
- 3) Update the weight vector according to the recursion:

$$\mathbf{g}_{n+1} = \mathbf{g}_n - \mu (y_{nQ+P-1} - y_{nQ+Q-1})^* \hat{\mathbf{r}}_n - \mu' \sum_{i=0}^{N-1} \bar{z}_n[i] \left( z_n^*[i] \mathbf{v}_n[i] - z_{n-1}^*[i] \mathbf{v}_{n-1}[i] \right) \quad (8)$$

where  $\bar{z}_n[i] = |z_n[i]|^2 - |z_{n-1}[i]|^2$ ,  $\hat{\mathbf{r}}_n = \hat{\mathbf{r}}_{nQ+P-1} - \hat{\mathbf{r}}_{nQ+Q-1}$ ,  $\mathbf{v}_n^T[i]$  is the  $i$ th row of  $\mathbf{V}_n$ , and  $\mu$  and  $\mu'$  are step gains.

- 4)  $n = n + 1$ , go back to Step 2).

The computational complexity of the algorithm is  $\mathcal{O}(N^2 + NL_t D)$ . When the weight vector approaches a desirable point where both IBI and IQ imbalance vanish, both  $\bar{z}_n[i]$  and  $y_{nQ+P-1} - y_{nQ+Q-1}$  in (8) become small. This results in small fluctuation of the weight vector in the final phase of the adaptation process.

It is noted that although the proposed algorithm is blind, it requires a few training symbols to obtain data symbol estimates at the receiver. After the TEQ vector  $\mathbf{g}$  is obtained, a total channel impulse response  $\mathbf{c}_1$  is required to build a frequency-domain equalizer. The total channel  $\mathbf{c}_1$  can be estimated blindly by existing methods such as [7]. An inherent problem of blind channel estimation methods is that the channel can only be estimated up to a complex scalar. A common technique to resolve this scalar ambiguity is to use a few training symbols (say, one or two symbols).

## 3. Simulation Results

We used the following simulation parameters: the DFT size  $N = 32$ , the CP length  $P = 8$ . In the case of the number of antennas  $D = 2$ , the transfer functions of filters  $f_{1,l}^{(d)}, f_{Q,l}^{(d)}$  of length  $L_r = 2$  in (1) were  $F_I^{(0)}(z) = 0.98 + 0.03z^{-1}$ ,  $F_I^{(1)}(z) = 0.95 - 0.02z^{-1}$ ,  $F_Q^{(0)}(z) = 1.0 - 0.005z^{-1}$ ,  $F_Q^{(1)}(z) = 1.01 + 0.007z^{-1}$ , and the gain mismatches and phase mismatches were set at  $[\gamma^{(0)} \ \gamma^{(1)}] = [1.05 \ 1.04]$  and  $[\phi^{(0)} \ \phi^{(1)}] =$

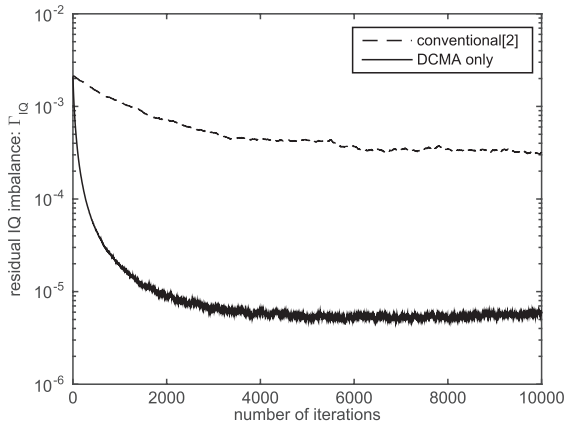


Fig. 1 Residual IQ imbalance.

$[\pi/60 \ \pi/61]$ . These filter parameters were chosen close to the values adopted in [3]. We considered quasi-static Rayleigh fading channels where the channel coefficients  $h_j^{(d)}$  were modeled as complex Gaussian random variables with zero mean and unit variance. The results were obtained by averaging over 100 independent trials. For BER simulations, we assumed the exact knowledge of the total channel  $\mathbf{c}_1$  after 5,000 iterations, and zero-forcing frequency-domain equalizer is built.

First, we show the effectiveness of the DCMA based IQ imbalance compensation. For comparison, a conventional method in [2] was also performed. We set the channel order  $L_h = 5$ , the TEQ length  $L_t = 3$ ,  $D = 1$ ,  $\mu = 0$ ,  $\mu' = 5 \times 10^{-7}$ , and  $\text{SNR} = 30\text{dB}$ .  $F_I^{(0)}$ ,  $F_Q^{(0)}$ ,  $\gamma^{(0)}$ , and  $\phi^{(0)}$  mentioned above were used. The step gain of the conventional method was  $[1 \ 0.5 \ 0.5] \times 10^{-5}$ . Fig. 1 shows the time evolution of the residual IQ imbalance defined by  $\Gamma_{\text{IQ}} = \|\mathbf{c}_2\|^2 / \|\mathbf{c}_1\|^2$ . It is clear from the figure that the performance of the DCMA is superior to the conventional method. As mentioned above, this is due to the small fluctuation of the weight vector.

Next, the BER performance of the proposed algorithm is shown in Fig. 2. We set  $D = 2$ ,  $L_h = 15$  and  $L_t = 16$ . We chose step gains, which provide the lowest BER for each SNR, from  $\mu \in \{10^{-3}, 10^{-4}, 10^{-5}\}$  and  $\mu' \in \{10^{-7}, 10^{-8}, 10^{-9}\}$ . We also show that of the receiver without compensation (labeled as “w/o compensation”), the receiver with only MERRY (MERRY only), the receiver with only DCMA (DCMA only). We can observe the effectiveness of the proposed joint compensation. There is a slight degradation from the case with no IBI and IQ imbalance ( $L_h = 8$ ). This degradation can be reduced if more iterations are allowed.

#### 4. Conclusion

In this letter, we presented a joint compensation algorithm

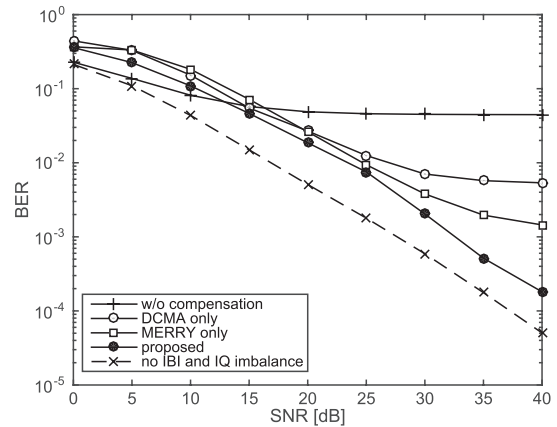


Fig. 2 BER performances.

for IBI and frequency-dependent IQ imbalance. Simulation results show that the proposed algorithm works well, and especially that DCMA is useful for the IQ imbalance compensation. A drawback of the proposed algorithm is a slow convergence. The development of a faster-converging algorithm is desired.

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