IEICE TRANS. FUNDAMENTALS, VOL.E96–A, NO.5 MAY 2013

LETTER

Joint Channel Shortening and Carrier Frequency Offset Estimation Based on Carrier Nulling Criterion in Downlink OFDMA Systems

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SUMMARY In this letter, we present a joint blind adaptive scheme to suppress inter-block interference and estimate a carrier frequency offset (CFO) in downlink OFDMA systems. The proposed scheme is a combination of a channel shortening method and a CFO estimator, both based on the carrier nulling criterion. Simulation results demonstrate the effectiveness of the proposed scheme.
key words: *block transmission, inter-block interference, blind equalizer, carrier frequency offset*

1. Introduction

Orthogonal frequency division multiple access (OFDMA) is a promising technique in multiuser wireless systems due to its high data rate and robustness to frequency-selective channels. Two critical issues in OFDMA systems are inter-block interference (IBI) suppression and carrier frequency offset (CFO) estimation. When the length of the channel impulse response exceeds that of cyclic prefix (CP), residual IBI results in severe performance degradation. An efficient technique to suppress IBI is channel shortening using a time-domain linear equalizer. In particular, blind channel shortening methods are attractive means for reducing transmission overhead due to training symbols [1], [2]. Along with IBI, OFDMA systems are also sensitive to CFO due to oscillator instabilities. CFO causes the loss of subcarrier orthogonality and introduces inter-carrier interference (ICI). So far, several blind CFO estimation methods have been developed [3]. In the most of previous studies, however, these two problems have been treated independently.

Recently, joint channel shortening and CFO estimation schemes for singleuser multicarrier systems have been proposed in [4] and [5], where the multicarrier equalization by restoration of redundancy (MERRY) algorithm [1] is employed as a channel shortening method. However, the performance of MERRY becomes unsatisfactory if the number of null (unused) subcarriers is larger than a threshold [6]. An alternative channel shortening scheme suitable for OFDMA systems where the number of null subcarriers can be large is the carrier nulling algorithm (CNA) [2] that is irrespective of the number of null subcarriers [6]. The cost function of CNA is the power of the DFT outputs corresponding to null subcarriers, i.e., carrier nulling criterion. Interestingly, the CNA cost function has been successfully applied to CFO estimation [3]. Thus, we can expect that the CNA [2] can be combined in a natural way with the CFO estimation scheme in [3]. In this letter, we propose a joint blind adaptive channel shortening and CFO estimation scheme, both based on the carrier nulling criterion, and show its effectiveness by computer simulation.

2. Joint Channel Shortening and CFO Estimation

2.1 Problem Formulation

We consider a downlink OFDMA system consisting of K users and N orthogonal subcarriers. Let $\mathcal{S}^{(k)}$ and $\mathcal{N}$ denote the index set of data subcarriers of the kth user and the index set of null subcarriers used by no one, respectively. The mth member of $\mathcal{S}^{(k)}$, denoted by $\mathcal{S}^{(k)}[m]$, is the mth data subcarrier index of the kth user. A data symbol vector of the kth user at the nth block is denoted by $\mathbf{s}_n^{(k)} = [s_n^{(k)}[0] \ \cdots \ s_n^{(k)}[|\mathcal{S}^{(k)}| - 1]]^T$ where $|\mathcal{S}^{(k)}|$ stands for the cardinality of a set $\mathcal{S}^{(k)}$ and $(\bullet)^T$ denotes transpose. At the transmitter, first, the data symbol vector is converted by inverse DFT to produce a time-domain block vector $\mathbf{x}_n^{(k)} = [x_n^{(k)}[0] \ \cdots \ x_n^{(k)}[N-1]]^T = \mathbf{F}^{(k)H}\mathbf{s}_n^{(k)}$ where $\mathbf{F}^{(k)}$ is an $|\mathcal{S}^{(k)}|\times N$ matrix whose mth row is the $\mathcal{S}^{(k)}[m]$th row of the N-point DFT matrix $\mathbf{F}$ whose (p,q)th entry is $\exp(-j2\pi pq/N)/\sqrt{N}$. Next, a cyclic prefix (CP) of length P is added to the beginning of $\mathbf{x}_n^{(k)}$ to form the nth transmitted block whose ith entry is $u_{nQ+i}^{(k)} = x_n^{(k)}[\mathrm{mod}(i+N-P, N)], i = 0, \cdots, Q-1$ where $Q = N + P$ is the OFDMA block length and $\mathrm{mod}(a, b)$ is the remainder of a modulo b.

We assume that a physical channel is modeled as FIR SIMO systems with D receive antennas. The channel from a base station to a terminal d has the impulse response $\{\mathbf{h}_i^{(d)}\}_{i=0}^{M}$ where $\mathbf{h}_i^{(d)}$ is a $D\times 1$ vector and M is the order of the channel. In the presence of CFO, the received signal at time l is

$$\mathbf{r}_l^{(d)} = e^{j\phi^{(d)}l} \sum_{m=0}^{M} \mathbf{h}_m^{(d)} \sum_{k=1}^{K} u_{l-m}^{(k)} + \mathbf{n}_l^{(d)} \ \in \mathbb{C}^{D\times 1} \tag{1}$$

where $\phi^{(d)}$ represents the CFO, and $\mathbf{n}_l^{(d)}$ is the noise vector. Note that the downlink OFDMA model in (1) includes a single-user OFDM system as a special case. Thus, the discussion to be presented in the rest of this letter holds also for a single-user OFDM system. For brevity, the superscript (d)

Manuscript received July 30, 2012.
Manuscript revised December 20, 2012.
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DOI: 10.1587/transfun.E96.A.1014

is omitted in the subsequent expressions.

If we have a CFO estimate $\hat{\phi}$, the CFO is compensated by $\hat{\mathbf{r}}_l = \mathbf{r}_l e^{-j\hat{\phi}l}$. When $M > P$, residual IBI occurs. To suppress IBI, we employ a time-domain linear multichannel equalizer. Each antenna output is filtered by an L-tap equalizer, and the D resulting outputs are added. Let $\mathbf{g} = [\mathbf{g}_0^H \ \cdots \mathbf{g}_{L-1}^H]^H$ where $\mathbf{g}_i \in \mathbb{C}^{D\times 1}$ be the equalizer coefficients vector and $(\bullet)^H$ denotes Hermitian transpose. At time l, the equalizer output is given by $y_l = \mathbf{g}^H \tilde{\mathbf{r}}_l$ where $\tilde{\mathbf{r}}_l = [\hat{\mathbf{r}}_l^T \ \cdots \ \hat{\mathbf{r}}_{l-L+1}^T]^T$. Removing CP and stacking N successive equalizer outputs, we have

$$\mathbf{y}_n = \begin{bmatrix} y_{nQ+P} \\ \vdots \\ y_{nQ+Q-1} \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{r}}_{nQ+P}^T \\ \vdots \\ \tilde{\mathbf{r}}_{nQ+Q-1}^T \end{bmatrix} \mathbf{g}^* = \hat{\mathbf{R}}_n \mathbf{g}^*. \qquad (2)$$

The equalizer output vector is transformed by DFT

$$\mathbf{z}_n = [z_n[0] \ \cdots \ z_n[N-1]]^T = \mathbf{F}\mathbf{y}_n. \qquad (3)$$

The DFT output $\mathbf{z}_n$ includes IBI and ICI in addition to the desired component. If the impulse response c_i of the effective channel, which is a total channel composed of the physical channel $\mathbf{h}_i$ and the equalizer $\mathbf{g}_i$, is shortened to within the CP length, residual IBI is suppressed completely, i.e., $c_i = 0$ for $i > P$ where $c_i = \sum_{m=0}^{M} \mathbf{h}_m^H \mathbf{g}_{i-m}, i = 0, \cdots, L+M-1$. Also, if CFO is compensated properly, ICI vanishes. Thus, the problem is to jointly shorten the effective channel and to compensate CFO by determining $\mathbf{g}$ and $\hat{\phi}$ blindly.

2.2 Proposed Scheme

When both IBI and/or CFO exist, there is some energy falling across the null subcarriers. The basic idea of the carrier nulling criterion is to minimize such energy. We consider the following cost function:

$$J_c(\mathbf{g}, \hat{\phi}) = E\left[\sum_{i\in\mathcal{N}} |z_n[i]|^2\right] = E\left[\|\mathbf{F}_{\mathrm{n}}\mathbf{y}_n\|^2\right] \qquad (4)$$

where $\mathbf{F}_{\mathrm{n}}$ is a $|\mathcal{N}| \times N$ matrix whose row vectors are the row vectors of $\mathbf{F}$, $\mathbf{f}_i^H, i \in \mathcal{N}$. We enforce the unit norm constraint $\|\mathbf{g}\| = 1$ to avoid $\mathbf{g} = \mathbf{0}$. It has been reported that, in the absence of IBI, CFO can be estimated correctly by minimizing the cost function [7]. Furthermore, it can be easily shown from the results in [6] that, in the absence of CFO, IBI can be suppressed by minimizing the cost function if

$$N > L + M - P - 1. \qquad (5)$$

In the presence of both CFO and IBI, though it is clear that a solution where both CFO and IBI are jointly compensated is a minimum of the cost function, it is not clear if the solution is unique. Strictly speaking, we need to analyze the surface of the cost function to clarify the uniqueness. However, it is not an easy task. Thus, at present, we can just expect that the harmful effect caused by IBI and CFO can be reduced by minimizing the cost function. In the following, we propose a practical method to minimize the cost function.

There could be several ways to minimize the cost function with respect to both the equalizer vector $\mathbf{g}$ and CFO estimate $\hat{\phi}$. In this letter, we employ a stochastic gradient algorithm due to its simplicity. The equalizer vector $\mathbf{g}[n]$ and CFO estimate $\hat{\phi}[n]$ are updated each time one OFDMA block is received where $[n]$ implies the nth iteration. The proposed procedure is as follows:

1) Initialize: $n = 0$, $\mathbf{g}[0] = [1\ 0 \cdots 0]^T$ and $\hat{\phi}[0] = 0$.
2) Compensate CFO by $\hat{\mathbf{r}}_l = \mathbf{r}_l e^{-j\hat{\phi}[n]l}$.
3) Obtain $\hat{\mathbf{R}}_n$ and $\mathbf{z}_n$ using Eqs. (2) and (3).
4) Update the equalizer weight vector and CFO estimation according to the recursions

$$\mathbf{g}[n+1] = \frac{\mathbf{g}[n] - \mu_1\left(\hat{\mathbf{R}}_n^T \mathbf{F}_{\mathrm{n}}^T \mathbf{F}_{\mathrm{n}}^* \hat{\mathbf{R}}_n^* \mathbf{g}[n]\right)}{\left\|\mathbf{g}[n] - \mu_1\left(\hat{\mathbf{R}}_n^T \mathbf{F}_{\mathrm{n}}^T \mathbf{F}_{\mathrm{n}}^* \hat{\mathbf{R}}_n^* \mathbf{g}[n]\right)\right\|}, \qquad (6)$$

$$\hat{\phi}[n+1] = \hat{\phi}[n] - \mu_2 \sum_{i\in\mathcal{N}} \mathrm{Re}\left\{\mathbf{f}_i^H \mathbf{q}_n z_n^*[i]\right\} \qquad (7)$$

where μ_1 and μ_2 are step sizes,

$$\mathbf{q}_n = \begin{bmatrix} \mathbf{g}^H[n]\mathbf{J}_{nQ+P}\tilde{\mathbf{r}}_{nQ+P} \\ \vdots \\ \mathbf{g}^H[n]\mathbf{J}_{nQ+Q-1}\tilde{\mathbf{r}}_{nQ+Q-1} \end{bmatrix} \qquad (8)$$

and

$$\mathbf{J}_m = \begin{bmatrix} -jm\mathbf{I}_D & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & -j(m-L)\mathbf{I}_D \end{bmatrix} \qquad (9)$$

where $\mathbf{I}_D$ is the $D \times D$ identity matrix.
5) $n = n + 1$, go back to Step 2).

The step sizes μ_1, μ_2 should be carefully chosen since they affect IBI suppression performance and CFO estimation accuracy as well as convergence rate.

The proposed method requires multiple DFT operations in (6) that originate in the original CNA. Though the total computational complexity of the proposed method is larger than that of MERRY, it is almost the same as that of the original CNA.

If the equalizer update and CFO estimate are performed separately as in a conventional way, we need to switch one to the other at appropriate timing that is unknown *a priori*. In contrast, in the proposed joint algorithm, we are no longer bothered by the switching timing.

3. Simulation Results

In computer simulation, the performance of the proposed scheme is compared with the conventional scheme in [4]. Simulation parameters were set as follows. The OFDMA DFT size $N = 64$, the CP length $P = 16$, the number of receive antennas $D = 2$, the channel order $M = 28$, the equalizer filter length $L = 29$, the number of users $K = 3$, and modulation scheme is QPSK. Channel coefficients were modeled as complex Gaussian random variables with

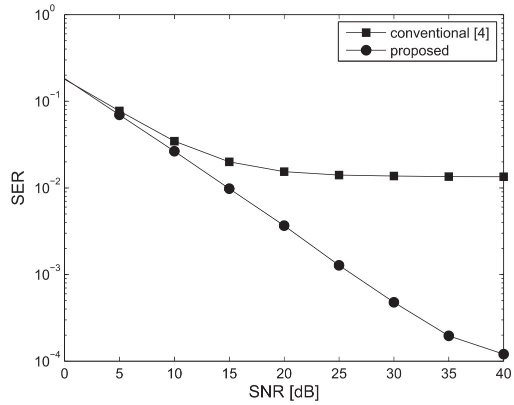

Fig. 1 SER performance comparison.

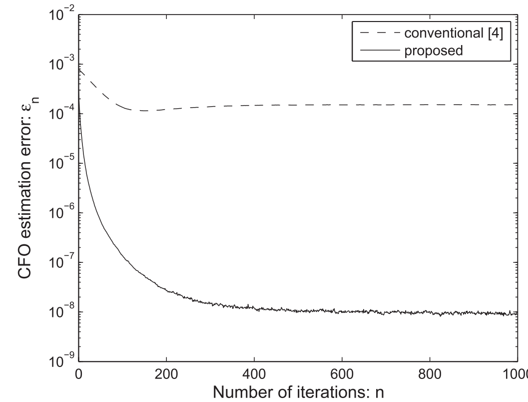

Fig. 2 Evolution of average CFO estimation error.

zero mean and variances $\sigma_m^2 = \lambda \exp(-\alpha m), m = 0, \cdots, M$ where $\alpha = 0.1$ and λ ensures the unit average energy of the channel. The CFO ϕ was uniformly distributed over $[-\pi/N, \pi/N]$. Each user used 10 subcarriers assigned randomly, and the remaining 34 subcarriers were null subcarriers. Receivers used a zero-forcing frequency domain equalizer built from the effective channel impulse response that was assumed to be known at the receiver. This is a common assumption in studies of channel shortening to evaluate channel shortening methods themselves by avoiding the effect of channel estimation errors. The step sizes, μ_1 for weight vector update and μ_2 for CFO estimate update, need to be chosen depending on communication conditions such as SNR. We chose a combination of step sizes, which provided the lowest SER, from $\mu_1 \in \{10^{-4}, 10^{-3}, 10^{-2}\}$ and $\mu_2 \in \{10^{-6}, 5 \times 10^{-5}, 10^{-5}, 10^{-4}\}$

A performance measure is the symbol error rate (SER), which is appropriate in digital wireless communication systems. At each trial, the steps 2)-5) were performed 1,000 iterations. After that, the obtained $\mathbf{g}[1000]$ and $\hat{\phi}[1000]$ were fixed and SER was computed by using 100 OFDMA blocks. We repeated the above computation 10,000 trials (different trials had different channels), and finally computed the average SER by averaging the SERs of each trial. Another performance measure is the CFO estimation error. At each trial and each iteration, we computed a CFO estimation error $(\phi - \hat{\phi}[n])^2$ by using $\mathbf{g}[n]$ and $\hat{\phi}[n]$. After 10,000 trials, we computed the average CFO estimation errors ε_n at each iteration by averaging the CFO errors of each trial.

Figure 1 shows SER performances in terms of SNR. The step sizes used by the proposed scheme were $(\mu_1, \mu_2) = (10^{-4}, 10^{-5})$ for SNRs 0, 5, and 10 dB; $(10^{-3}, 5 \times 10^{-5})$ for SNRs 15 and 20 dB; $(10^{-2}, 10^{-4})$ for SNR 25 dB; $(10^{-2}, 5 \times 10^{-5})$ for SNRs 30, 35, and 40 dB, those used by the conventional scheme were $(\mu_1, \mu_2) = (10^{-4}, 10^{-5})$ for all SNRs. As we can see, the proposed scheme is superior to the conventional one especially in high SNR region.

Figure 2 illustrates the time evolution of the average of CFO estimation errors $\epsilon_n = (\phi - \hat{\phi}_n)^2$ where SNR=30 dB and the step sizes were the same as in Fig. 1. It can be seen from the figure that the CFO estimation error is significantly reduced by the proposed scheme. It is generally known that stochastic gradient algorithms are simple but have a drawback of slow convergence rate. Thus, a development of fast and simple algorithms is worth considering.

4. Conclusion

We have proposed a joint adaptive channel shortening and CFO estimation scheme based on the carrier nulling criterion. Simulation results have demonstrated that the proposed scheme can significantly combat the harmful effects of IBI and CFO.

As mentioned in Sect. 2.2, an important future issue is cost function surface analysis. Also, an extension to uplink OFDMA systems where multiple access interference should be taken into account is a future challenging issue. Furthermore, one could investigate a semi-blind joint shortening and CFO estimation method for the case where pilot symbols are available.

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