LETTER *Special Section on Wideband Systems*

A Max-Min Approach to Channel Shortening in OFDM Systems

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SUMMARY In OFDM systems, residual inter-block interference can be suppressed by a time-domain equalizer that blindly shortens the effective length of a channel impulse response. To further improve the performance of blind equalizers, we propose a channel shortening method that attempts to maximize the minimum FFT output power over data subcarriers. Simulation results indicate that the max-min strategy has performance improvement over a conventional channel shortening method.

key words: *inter-block interference, carrier nulling algorithm, blind equalization, semi-definite relaxation*

1. Introduction

Orthogonal frequency division multiplexing (OFDM) with cyclic prefix (CP) is a promising technique for high speed transmissions over frequency selective channels. When the length of channel impulse response exceeds the CP length, however, residual inter-block interference (IBI) degrades the system performance. A bandwidth efficient way to mitigate the effect of IBI is blind channel shortening using a time-domain equalizer (TEQ) [1], such as carrier nulling algorithm (CNA) [2] and MERRY algorithm [3]. It has been proven that these blind channel shortening methods can perfectly suppress IBI by shortening the impulse response of the overall effective channel consisting of a physical channel and a TEQ to within the CP length [4], [5]. It should be noted, however, that the resulting effective channel is still frequency selective. Consequently, some subcarriers of the effective channel can be subject to a deep fading that results in degraded performance.

In this letter, we propose an improvement of CNA. In the proposed method, deep fading is reduced by maximizing the FFT output power per data subcarrier. More specifically, it maximizes the minimum power of FFT outputs corresponding to data subcarriers while keeping the total power of FFT outputs corresponding to null subcarriers minimal. The constrained min-max problem is solved by semi-definite relaxation [6], [7]. The effectiveness of the proposed method is verified by simulation.

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2. OFDM System with TEQ

We consider an OFDM system with N subcarriers. Let I_d and I_n stand for the data subcarrier index set and null subcarrier index set, respectively. The nth data block is denoted as $\mathbf{s}_n = \left[\, s_n[0] \cdots s_n[N-1]\,\right]^T$ where $s_n[i] = d_n[i]$ for $i \in I_\mathrm{d}$, $s_n[i] = 0$ for $i \in I_\mathrm{n}$, and $d_n[i]$ is a symbol sequence with variance σ_d^2. Applying IFFT to $\mathbf{s}_n$, we get $\mathbf{x}_n = \mathbf{F}^H\mathbf{s}_n = \left[\, x_n[0] \cdots x_n[N-1]\,\right]^T$ where $\mathbf{F}$ is the FFT matrix. A CP of length P is added to the beginning of $\mathbf{x}_n$ to form the nth transmitted block $\mathbf{u}_n$ whose ith entry is $u_{nQ+i} = x_n[(i+N-P)_N]$ where $i = 0, 1, \cdots, Q-1$, $Q = N+P$, and $(m)_N$ represents the reminder of m modulo N. A channel is assumed to be quasi-static. The signal received by D antennas is given by

$$\tilde{\mathbf{r}}_k = \sum_{m=0}^{M} \mathbf{h}_m u_{k-m} + \mathbf{w}_k \ \in \mathbb{C}^{D\times 1} \tag{1}$$

where $\mathbf{h}_m = [h_m[0] \ \cdots \ h_m[D-1]]^T \in \mathbb{C}^{D\times 1}$ represents an $M+1$ taps long impulse response vector of a SIMO FIR channel and $\mathbf{w}_k$ is a noise vector.

The output of TEQ of length L is given by $y_k = \mathbf{g}^H\mathbf{r}_k$ where $\mathbf{r}_k = \left[\, \tilde{\mathbf{r}}_k^T \cdots \tilde{\mathbf{r}}_{k-L+1}^T\,\right]^T \in \mathbb{C}^{LD\times 1}$ and $\mathbf{g} \in \mathbb{C}^{LD\times 1}$ is the impulse response vector of the TEQ. Removing CP and collecting the TEQ outputs for the nth block, we have

$$\mathbf{y}_n = \begin{bmatrix} y_{nQ+P} \\ \vdots \\ y_{nQ+Q-1} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{r}_{nQ+P}^T \\ \vdots \\ \mathbf{r}_{nQ+Q-1}^T \end{bmatrix}}_{\mathbf{R}_n} \mathbf{g}^*. \tag{2}$$

Then, we have the FFT output as $\mathbf{z}_n = \mathbf{F}\mathbf{y}_n$. The purpose of blind channel shortening is to suppress IBI by properly adjusting TEQ $\mathbf{g}$ without training symbols.

3. Channel Shortening

Here, we focus on CNA [2]. When IBI exists, the IBI component appears at the FFT outputs corresponding to null subcarriers. Thus, to suppress the IBI component, CNA minimizes the FFT output power corresponding to null subcarriers. The cost function of CNA is given by

$$J(\mathbf{g}) = E\left[\sum_{i\in I_\mathrm{n}} |z_n[i]|^2\right] \tag{3}$$

where $z_n[i]$ is the ith entry of $\mathbf{z}_n$. Since $z_n[i]$ is given by $z_n[i] = \mathbf{f}_i^H \mathbf{R}_n \mathbf{g}^*$ where $\mathbf{f}_i^H$ is the ith row of $\mathbf{F}$, the cost function can be rewritten as

$$J(\mathbf{g}) = \sum_{i \in I_\mathrm{n}} \mathbf{g}^T \mathbf{R}_{ii} \mathbf{g}^* \tag{4}$$

where $\mathbf{R}_{ii} = E[\mathbf{R}_n^H \mathbf{f}_i \mathbf{f}_i^H \mathbf{R}_n]$. If the norm constraint $\|\mathbf{g}\|^2 = 1$ is imposed, the complex conjugate of the eigenvector corresponding to the minimum eigenvalue of $\sum_{i \in I_\mathrm{n}} \mathbf{R}_{ii}$ is the optimal solution and can be used as the TEQ vector $\mathbf{g}$. It has been shown that the effective channel $\mathbf{c} = \mathbf{H}^H \mathbf{g} \in \mathbb{C}^{(L+M)\times 1}$ can be shortened, i.e., IBI is suppressed perfectly, by minimizing J, where $\mathbf{H} \in \mathbb{C}^{LD\times(L+M)}$ defined by

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}_0 & \cdots & \mathbf{h}_M & & \mathbf{0} \\ & \ddots & & \ddots & \\ \mathbf{0} & & \mathbf{h}_0 & \cdots & \mathbf{h}_M \end{bmatrix}$$

is assumed to be tall [4].

Subcarriers in the resulting effective channel $\mathbf{c}$ can be in a deep fading. To overcome this problem, we consider maximizing the FFT output power over data subcarriers under the condition that a channel is shortened. More precisely, we propose to maximize the power of FFT output corresponding to the worst data subcarrier $\mathbf{g}^T \mathbf{R}_{ii} \mathbf{g}^*$ ($i \in I_\mathrm{d}$) to compensate for deep fading, while keeping the power of FFT outputs corresponding to null subcarriers $\mathbf{g}^T \mathbf{R}_{jj} \mathbf{g}^*$ ($j \in I_\mathrm{n}$) minimal to shorten a channel.

Now, the proposed method can be represented as

$$\begin{aligned} &\max_{\mathbf{g}} \min_{i \in I_\mathrm{d}} \mathbf{g}^T \mathbf{R}_{ii} \mathbf{g}^* \\ &\text{s.t. } \mathbf{g}^T \mathbf{R}_{jj} \mathbf{g}^* = 0 \ (j \in I_\mathrm{n}). \end{aligned} \tag{5}$$

In the presence of the channel noise, the minimum of null subcarrier outputs power is nonzero. We replace the minimum to zero in the above constraints because it is difficult to know the minimum in advance. To solve the problem efficiently, we employ the approach in [7]. Defining a positive semidefinite matrix $\mathbf{G} = \mathbf{g}^* \mathbf{g}^T$ of rank 1, we can recast the problem into

$$\begin{aligned} &\max_{\mathbf{G}} \min_{i \in I_\mathrm{d}} \mathrm{trace}(\mathbf{G}\mathbf{R}_{ii}) \\ &\text{s.t. } \mathrm{trace}(\mathbf{G}\mathbf{R}_{jj}) = 0 \ (j \in I_\mathrm{n}), \\ &\qquad \mathrm{rank}(\mathbf{G}) = 1, \ \mathbf{G} \geq 0 \end{aligned} \tag{6}$$

where $\mathbf{G} \geq 0$ implies that $\mathbf{G}$ is positive semidefinite. Generally, problems with a non-convex rank-one constraint are difficult to solve. One approach to solve this problem approximately is to apply semidefinite relaxation by dropping the rank-one constraint. Introducing nonnegative slack variables b_i and additional variable t, the above problem can be converted into a convex semidefinite programming (SDP) in a similar way as in [7]:

$$\begin{aligned} &\min \ -t \\ &\text{s.t. } -t - b_i + \mathrm{vec}(\mathbf{R}_{ii}^T)^T \mathrm{vec}(\mathbf{G}) = 0 \ (i \in I_\mathrm{d}), \\ &\qquad \mathrm{vec}(\mathbf{R}_{jj}^T)^T \mathrm{vec}(\mathbf{G}) = 0 \ (j \in I_\mathrm{n}), \\ &\qquad \mathbf{G} \geq 0, \ t \geq 0, \ b_i \geq 0 \end{aligned} \tag{7}$$

where $\mathrm{vec}(\mathbf{X})$ denotes the vector obtained by stacking the columns of $\mathbf{X}$. This can be solved by SDP solvers such as SeDuMi. In our simulation, after we got a solution $\mathbf{G}$, we chose the eigenvector corresponding to the maximum eigenvalue of $\mathbf{G}$ as $\mathbf{g}^*$.

4. Simulation Results

Simulation parameters are set as follows: $N = 32, P = N/4 = 8, D = 2$, QPSK modulation scheme, and three subcarriers placed at each edge of allocated bandwidth and a direct current subcarrier are nulled. We choose $M = 14$ such that IBI occurs. Also, we set $L = 15$ such that $\mathbf{H}$ becomes tall. All channel tap coefficients $h_m[d]$ are modeled as zero-mean complex Gaussian random variables with variances, $\sigma_m^2 = \lambda \exp(-\alpha m), m = 0, \cdots, M$, where $\alpha = 0.1$ and λ ensures the unit average energy of the channel. The correlation matrices $\mathbf{R}_{ii}$ are estimated by time-averaging over 100 blocks, that were enough to provide satisfactory results in our preliminary simulations.

A performance measure is SNR per data subcarrier at the FFT outputs defined by

$$\gamma_i = \frac{\sigma_d^2 \mid H_c(e^{\frac{j2i\pi}{N}}) \mid^2}{E\left[\mid \mathbf{f}_i^H \mathbf{W}_n \mathbf{g}^* \mid^2\right]}, \ i \in I_\mathrm{d} \tag{8}$$

where $H_c(e^{\frac{j2i\pi}{N}})$ is the frequency gain of the effective channel and $\mathbf{W}_n$ is the noise component in $\mathbf{R}_n$. Figure 1 illustrates an example of SNR per data subcarrier where received SNR is 30 dB. Though SNRs of the proposed method are slightly worse than those of CNA at some subcarriers, this degradation is not serious as long as the degradation is small. Rather, it should be emphasized that the proposed method can prevent deep spectral nulls that degrade the performance of CNA seriously.

BER performance of the proposed method, CNA, and the receiver without TEQ is compared in Fig. 2 where BER

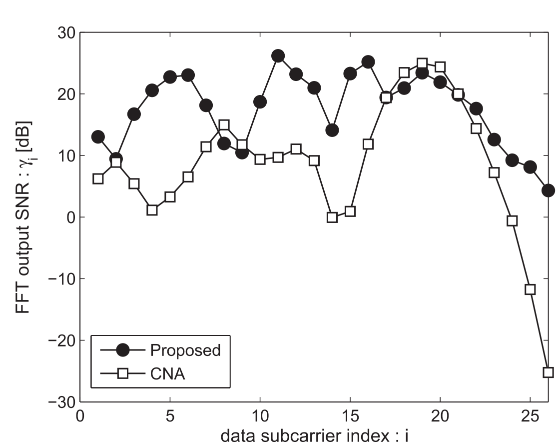

Fig. 1 FFT output SNR per data subcarrier.

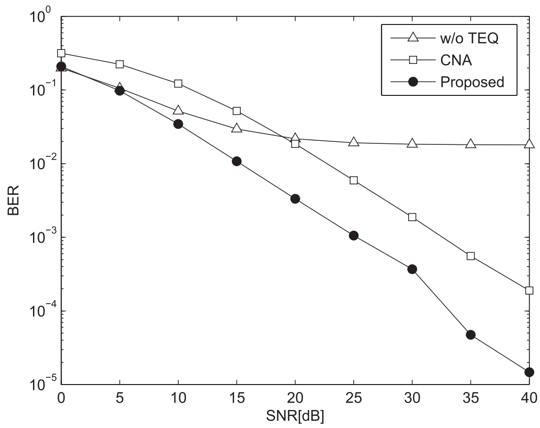

Fig. 2 BER performance comparison.

is obtained by averaging over 10,000 blocks and 1,000 different trials, and assuming that the knowledge of the shortened channel for frequency-domain equalization. The performance of CNA is inferior to that of the receiver without TEQ in low SNR regions. This is because of noise enhancement caused by the deep spectral nulls as seen in Fig. 1. On the other hand, the proposed method can avoid such deep nulls and provide superior performance to both CNA and the receiver without TEQ in all SNR regions.

5. Conclusion

In this letter, we propose an improved channel shortening method to compensate for spectral nulls in the effective channel. Simulation results show that the performance of CNA is improved by the proposed method.

The constraint we used in (5) might not be an optimal choice. A more effective way to set the constraint will be explored. Also, developing a shortening method that requires only a few blocks is a challenging issue.

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