LETTER *Special Section on Signal Processing*

Interference Suppression Capability of Array Antenna Using Cyclic Prefix in Block Transmission

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SUMMARY In block transmission systems, performance degrades due to inter-block interference (IBI) when there are multipaths with delays exceeding cyclic prefix (CP) length. An interesting technique to overcome this problem is an array antenna proposed by Hori et al., which restores the CP property by minimizing a cost function. However, its performance has not been theoretically cleared. In this letter, the performance of a method which minimizes the cost function under a unit norm constraint is analyzed. It is shown that the method can suppress IBI and its interference suppression capability depends on a certain parameter. The analytical result is verified through computer simulation.

key words: *blind adaptive array antenna, inter-block interference, OFDM*

1. Introduction

Block transmission systems such as orthogonal frequency division multiplexing (OFDM) is a popular method for high-rate data transmission and has been successfully applied to wide variety of wireless systems such as digital broadcasting and wireless LAN. In block transmission systems, cyclic prefix (CP) is inserted between transmitted blocks to prevent inter-block interference (IBI), so that the beginning part of a block is equal to the ending of the block. However, when there are multipaths with delays exceeding CP length, residual IBI severely degrades the system performance.

To overcome the disadvantage, there are several approaches such as interference cancellation [1], time-domain equalization [2], and adaptive array antenna [3], [4]. As for adaptive array antenna, there is a recent interesting work by Hori et al. [5]. They focused on the fact that the CP property observed in the transmitted signal is destroyed in the received signal due to IBI. To restore the CP property at the array output, they employed a cost function which is the mean squared difference between an array output and its delayed version. A remarkable feature of the method is that, unlike training-based methods [4], it can be implemented by blind algorithms. In [5], the sample matrix inversion (SMI) technique was used to obtain an array weight vector which hopefully minimizes the cost function. Although the effectiveness of the method was verified by simulation, its interference suppression capability has not been analyzed yet. The performance analysis of the SMI based method is difficult because of the nonstationarity of the transmitted OFDM signals.

Imposing a unit norm constraint on the weight vector to avoid a trivial solution makes the minimization problem tractable. In this letter, thus, the performance of a method which minimizes the cost function proposed in [5] under the unit norm constraint is theoretically analyzed. It is shown that its interference suppression characteristics can be controlled by a certain parameter.

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2. Analysis of Array Antenna Using CP

2.1 Communication Model

In block transmission system using CP of length P, the baseband transmitted signal can be represented as

$$u(t) = \sum_{j} \sum_{i=0}^{Q-1} u_{jN+(i+N-P)_N} c(t - jT_Q - iT_s) \tag{1}$$

where $\{u_{jN+k}\}_{k=0}^{N-1}$ is the jth (transformed) symbol sequence of length N and has a variance σ_u^2, $c(t)$ is the pulse waveform of width T_c, $Q = P + N$ is the block length, T_s is the sampling period, $T_Q = QT_s$ is the block duration, and $(i)_N$ is the residue of i modulo N. Due to the CP property, $u(t) = u(t + T_N)$ for $jT_Q \leq t \leq jT_Q + T_P$, where $T_P = PT_s$ is the CP duration.

Let us consider a communication system where there are $M + 1$ paths, i.e., a direct wave and M delayed waves. The signal vector received by a uniform linear array composed of K antennae with half wavelength element spacing is described by the following model

$$\mathbf{x}(t) = \mathbf{A}\mathbf{u}(t) + \mathbf{n}(t) \tag{2}$$

where $\mathbf{A} = [h_0\mathbf{a}_0 \cdots h_M\mathbf{a}_M]$, h_i is the ith path gain, $\mathbf{a}_i = \left[e^0 \cdots e^{j\pi(K-1)\sin\theta_i}\right]^T$, θ_i is the impinging direction of the arriving wave from the ith path, $\mathbf{u}(t) = [u(t-\tau_0) \cdots u(t-\tau_M)]^T$, $\tau_0 < \cdots < \tau_M$ is the delay of each arriving wave, $\mathbf{n}(t) = [n_1(t) \cdots n_K(t)]^T$ is a noise vector, and $(\cdot)^T$ represents the transpose of a matrix. The channel noise $n_k(t)$ is assumed to be spatially and temporally white Gaussian with zero mean and variance equal to σ_n^2 and uncorrelated with the transmitted signal. The array antenna output is given by

$$y(t) = \mathbf{w}^H\mathbf{x}(t) \tag{3}$$

where $\mathbf{w}$ is the array weight vector and $(\cdot)^H$ represents the Hermitian transpose of a matrix.

2.2 Cost Function

If IBI component at the array output does not exist, there exists t such that $y(t)$ is equal to $y(t + T_N)$ due to the CP property. Based on the fact, a cost function was introduced [5]

$$J(\mathbf{w}) = E[|y(\delta) - y(\delta + T_N)|^2] \quad (4)$$

where $\delta \in [0, T_Q]$ is the delay parameter. In [5], the SMI technique was used to avoid a trivial solution $\mathbf{w} = \mathbf{0}$. It is not clear, however, the solution obtained by the SMI technique minimizes the cost function. Here, we impose a unit norm constraint on $\mathbf{w}$. The cost function can be rewritten as

$$J(\mathbf{w}) = E[|\mathbf{w}^H \tilde{\mathbf{x}}_\delta[0]|^2] = \mathbf{w}^H \mathbf{R}_x \mathbf{w}, \quad \text{s.t. } \|\mathbf{w}\| = 1 \quad (5)$$

where $\tilde{\mathbf{x}}_\delta[k] = \mathbf{x}(kT_Q + \delta) - \mathbf{x}(kT_Q + \delta + T_N)$, $\mathbf{R}_x = E[\tilde{\mathbf{x}}_\delta[0]\tilde{\mathbf{x}}_\delta^H[0]]$, and $\|\cdot\|$ is l_2 norm. Thus, J can be minimized by the eigenvector corresponding to the minimum eigenvalue of $\mathbf{R}_x$.

2.3 Effect of Delay Parameter on Performance

In [6], to control which arriving waves are suppressed, a window function was introduced. In the following, we show that the control is readily possible by appropriately setting the delay parameter δ.

Theorem 1: If $\mathbf{A}$ has full column rank, $\mathbf{R}_u = 2\sigma_u^2\mathbf{I}$, where $\mathbf{R}_u = E[\tilde{\mathbf{u}}_\delta\tilde{\mathbf{u}}_\delta^H]$ and $\tilde{\mathbf{u}}_\delta = \mathbf{u}(\delta) - \mathbf{u}(\delta + T_N)$, and $\delta \in [0, T_Q]$ is given, then the arriving waves whose delay τ_i does not satisfy $\delta - T_P < \tau_i < \delta$ can be suppressed by minimizing the cost function J.

Proof: All the arriving waves are classified into two sets: $\mathcal{A} = \{i | \delta - T_P < \tau_i < \delta\}$ and its complement $\bar{\mathcal{A}}$. For the $i \in \mathcal{A}$ th wave, $\tilde{u}(\delta - \tau_i) = u(\delta - \tau_i) - u(\delta + T_N - \tau_i) = 0$. Then, the cost function can be rewritten as the summation of the IBI and noise components

$$\begin{aligned} J(\mathbf{w}) &= E\left[\left|\mathbf{w}^H \sum_{i \in \bar{\mathcal{A}}} \mathbf{a}_i \tilde{u}(\delta - \tau_i)\right|^2\right] + 2\sigma_n^2 \\ &= \mathbf{w}^H \bar{\mathbf{A}} \bar{\mathbf{R}}_u \bar{\mathbf{A}}^H \mathbf{w} + 2\sigma_n^2 \end{aligned} \quad (6)$$

where $\bar{\mathbf{A}} = \left[\mathbf{a}_{\iota(1)} \cdots \mathbf{a}_{\iota(|\bar{\mathcal{A}}|)}\right]$, $\iota(k)$ is the kth smallest element in $\bar{\mathcal{A}}$, $|\bar{\mathcal{A}}|$ is the number of the elements of $\bar{\mathcal{A}}$, $\bar{\mathbf{R}}_u = E[\bar{\mathbf{u}}_\delta \bar{\mathbf{u}}_\delta^H]$, and $\bar{\mathbf{u}}_\delta = \left[\tilde{u}(\delta - \tau_{\iota(1)}) \cdots \tilde{u}(\delta - \tau_{\iota(|\bar{\mathcal{A}}|)})\right]$. Since it is clear from a given condition that $\bar{\mathbf{R}}_u = 2\sigma_u^2\mathbf{I}$, then $J(\mathbf{w}) = 2\sigma_u^2 \mathbf{w}^H \bar{\mathbf{A}} \bar{\mathbf{A}}^H \mathbf{w} + 2\sigma_n^2$. Since $\mathbf{A}$ has full column rank, $\bar{\mathbf{A}}$ has full column rank. Thus, $\bar{\mathbf{A}}^H \mathbf{w} = 0$ has a nontrivial solution $\mathbf{w}_0$. Then, the cost function is minimized and its minimum becomes $J(\mathbf{w}_0) = 2\sigma_n^2$, this implies that the arriving waves $u(t - \tau_i)$ for $i \in \bar{\mathcal{A}}$ are nulled. □

Let us mention about the conditions in the theorem. Since $\mathbf{A}$ is a Vandermonde matrix, if $\theta_i \neq \theta_j$ for $i \neq j$ and $M + 1 \leq K$, $\mathbf{A}$ has full column rank. Furthermore, it is easily confirmed that $\mathbf{R}_u = 2\sigma_u^2\mathbf{I}$, if u_k is white, $\tau_{i+1} - \tau_i > T_c$ for all i, and $\tau_M - \tau_0 < T_N - T_c$.

According to the theorem, we can suppress any arriving waves beyond a T_P interval, in other words IBI can be always suppressed by minimizing J. Also, by appropriately setting δ, we can control which arriving waves are suppressed. Let us show two examples: (i) when $\delta = T_P$ only the arriving waves whose delay satisfies $0 < \tau_i < T_P$ are extracted, (ii) when $\tau_0 < \delta < \tau_1$ all waves are suppressed except the direct wave.

3. Simulation Results

To verify the above analytical results, we carried out computer simulation. A batch processing was employed to obtain an array weight vector. First the correlation matrix $\mathbf{R}_x$ was estimated by time-averaging $\hat{\mathbf{R}}_x = 1/B \sum_{j=0}^{B-1} \mathbf{x}_\delta[j]\mathbf{x}_\delta^H[j]$, and then the eigenvector corresponding to the minimum eigenvalue of $\hat{\mathbf{R}}_x$ was computed and used as an array weight vector.

In the following simulation, we set $B = 1000, N = 16, T_P = 4T_s$, SNR=20[dB], and the path gains h_i were zero-mean complex Gaussian random variables. There were four arriving waves, i.e. $M = 3$. Their delay and direction were $\tau_0 = 0$, $\tau_1 = 2T_s$, $\tau_2 = 7T_s$, $\tau_3 = 8T_s$, and $\theta_0 = -20, \theta_1 = 60, \theta_2 = -60, \theta_3 = 20$, respectively.

In Fig. 1, beam patterns for $\delta = T_P$ and T_s are shown where $K = 4$. In the case of $\delta = T_P$, the first and second waves are extracted. On the other hand, only the direct wave is extracted in the case of $\delta = T_s$. These results are coincident with our analysis.

Figure 2 shows the magnitude of the array response to the arrival waves defined by $V_i = |\mathbf{w}^H \mathbf{a}(\theta_i)|$ when the delay parameter δ varies.In this case,we set $K = 5$. It can be observed that the array response depends on δ and the arriving waves which does not satisfy the condition $\delta - T_P < \tau_i < \delta$ are suppressed.

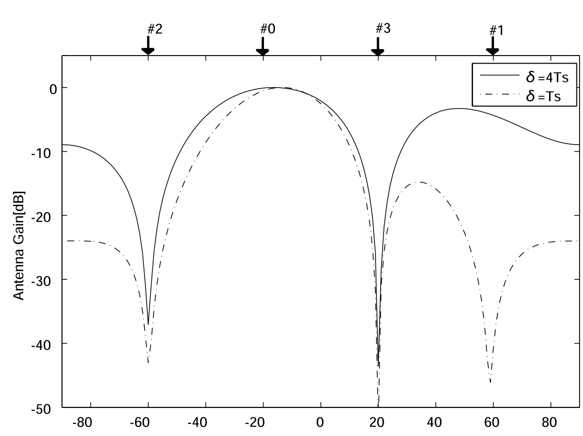

Fig. 1 Beam patterns for different δ.

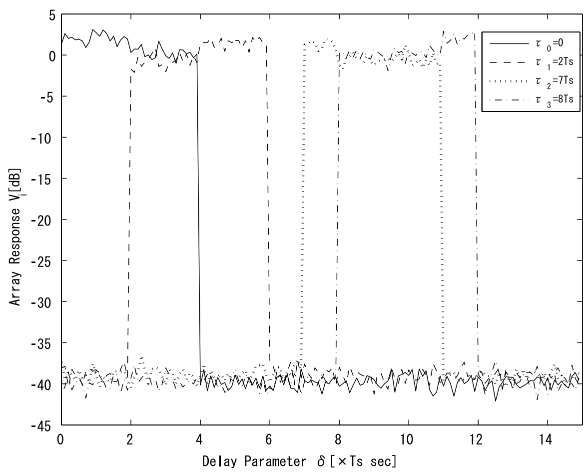

Fig. 2 Array responses to arriving waves controlled by varying delay parameter δ.

4. Conclusion

In this letter, the performance of the method based on the cost function proposed in [5] has been analyzed. It was shown that IBI can be suppressed by minimizing the cost function and the delay parameter can control which arriving waves are suppressed. The validity of the analytical result was verified by the simulation results of the method implemented by the batch processing. An adaptive algorithm for time-domain equalizers based on a similar cost function has been proposed in [2] and its interference suppression capability has been analyzed in [7]. Though the approach in [7] is a good reference, the result presented in this letter is not directly derived from it.

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