PAPER *Special Section on Wideband Systems*

Blind Channel Shortening for Block Transmission of Correlated Signals

Teruyuki MIYAJIMA[†a)] ***and*** **Yoshihisa WATANABE**[††*], ***Members***

SUMMARY In block transmission systems, blind channel shortening methods are known to be effective to reduce the influence of interblock interference which degrades the performance when the length of a channel impulse response is extremely long. Conventional methods assume that the transmitted signal is uncorrelated; however, this assumption is invalid in practical systems such as OFDM with null carriers and MC-CDMA. In this paper, we consider blind channel shortening methods for block transmissions when the transmitted samples within a block are correlated. First, the channel shortening ability of a conventional method is clarified. Next, a new method which exploits the fact that the transmitted samples in different blocks are uncorrelated is introduced. It is shown that the proposed method can shorten the channel properly under certain conditions. Finally, simulation results of OFDM and MC-CDMA systems are shown to verify the effectiveness of the proposed method compared with a conventional one.

key words: *OFDM, OFDMA, MC-CDMA, cyclic prefix, time-domain equalizer, inter-block interference*

1. Introduction

Block transmission techniques such as OFDM [1], multicarrier (MC)-CDMA [2], and single-carrier transmission with frequency domain equalization [3] is becoming very popular for high-speed communications especially in wireless environments and have been adopted in numerous standards [4]. In block transmission systems, inter-block interference (IBI) caused by time-dispersive channels can be avoided as long as the length of cyclic prefix (CP) which is inserted between data blocks is sufficiently longer than that of the channel impulse response. However, in the case where the channel impulse response is extremely long, the use of a longer CP causes the reduction in bandwidth efficiency. Such a situation can be found in many applications such as digital subscriber line communications [5], high-definition television broadcasting [6], and wireless LAN [7]. To avoid using a long CP, a number of approaches have been explored in the last decade, for example, decision feedback equalization [8], interference cancellation [6], adaptive array antenna [9], [10], and per-tone equalization [11]. However, these existing approaches have their own merits and drawbacks, the decisive approach has not been established yet.

Manuscript received March 5, 2008.
Manuscript revised June 6, 2008.

[†]The author is with the Domain of Systems Engineering, Ibaraki University, Hitachi-shi, 316-8511 Japan.
[††]The author is with the Graduate School of Science and Engineering, Ibaraki University, Hitachi-shi, 316-8511 Japan.
[*]Presently, with Mitsubishi Electric Corporation.
a) E-mail: miyajima@mx.ibaraki.ac.jp

DOI: 10.1093/ietfec/e91–a.11.3095

This paper focuses on the channel shortening approach which can be viewed as a generalization of linear equalization using an FIR filter [5], [12]. This tries to shorten the impulse response of the effective channel, which is a convolution of a channel and an equalizer, to a prescribed length. A distinguishable feature of the approach compared to the others is that its performance is tractable because of the linearity of the system. Channel shortening methods were originally developed for maximum likelihood sequence estimation and later studied extensively for DSL communications. In these studies, a channel impulse response is assumed to be estimated by means of training sequences.

More recently, focus has shifted to blind channel shortening methods which have high spectral efficiency because it requires no training sequences [13]–[23]. The pioneering work is the Multicarrier Equalization by Restoration of RedundancY (MERRY) algorithm [14], which aims to restore the CP property at time-domain equalizer (TEQ) output and can be implemented by both batch and adaptive algorithm. Some variants of the MERRY algorithm have been presented [15]–[17]: a modified algorithm was shown to be superior to the original MERRY algorithm in terms of TEQ output SINR [17]. Other methods relied on the second-order statistics (SOS) of the received signals perform well at the expense of computational complexity [18], [19]. A less complex SOS method based on the minimum mean output energy criterion can deal with narrowband interference as well [20]. Higher-order statistical methods have been presented: the sum-squared autocorrelation minimization (SAM) algorithm [21] and its simplified version [22] are adaptive but not global convergent.

In all the above existing methods, it was assumed that the transmitted time-domain samples before the CP insertion are uncorrelated. However, in many practical situations,this assumption is invalid. In OFDM systems such as IEEE 802.11a [4], the transmitted samples before the CP insertion are temporally correlated due to the existence of the null carriers making a guard band. A similar situation arises in OFDMA systems [24] where each user uses a subset of carriers and the remaining carriers can be viewed as null carriers. Another typical example is an MC-CDMA system where all carriers convey the same information symbol [2]. To the best of our knowledge, there is only one study on blind channel shortening dealing with such temporally correlated samples [23]. However, this approach is impractical since the frequency hopping of null carriers is assumed. It is pointed out in [13] that temporally correlated transmitted samples may degrade performance. Thus, a new blind channel shortening method has been desired to deal with temporally correlated samples.

In this paper, first we clarify the limitation of the conventional MERRY algorithm. It is shown that MERRY fails to shorten a channel in both OFDM with null carriers and MC-CDMA since the correlation matrix of highly correlated samples becomes rank-deficient. Then, we propose a new blind channel shortening method which is based on the fact that a time-domain sample is uncorrelated with one in a consecutive block even if it is correlated with another ones in the same block, and analyze its shortenability when applied to both OFDM with null carriers and MC-CDMA system. The proposed method computes a TEQ directly from the second-order statistics of the received signal without channel estimation. Simulation results are presented to demonstrate the superiority of the proposed algorithm compared to the conventional one.

The rest of the paper is organized as follows. In Sect. 2, we first describe a block transmission model and then describe the performance of the MERRY algorithm in Sect. 3. Section 4 presents a new method and discusses the applicability to practical systems. Simulation results are shown in Sect. 5.

2. Problem Formulation

We consider a single-user SIMO block transmission system with multiple antennae and an equalizer at the receiver as shown in Fig. 1. Let the ith time-domain sample block of length N before the CP insertion be $\mathbf{x}_i = [x_0[i] \cdots x_{N-1}[i]]^T$ where the superscript T represents the transpose of a matrix. Throughout the paper, the correlation property of the time-domain samples is assumed to be

$$E\left[x_n[i]\, x^*_{n'}[i']\right] = \begin{cases} \phi_{nn'} & i = i' \\ 0 & i \neq i' \end{cases} . \tag{1}$$

This assumption means that a time-domain sample $x_n[i]$ is correlated with another one $x_{n'}[i]$ in the same block but uncorrelated with ones $x_{n'}[i']$ in a different block. In multicarrier systems, the time-domain block can be obtained by $\mathbf{x}_i = \mathbf{F}\mathbf{s}_i$ where $\mathbf{F}$ is a unitary IFFT matrix whose (l, l')th element is $\exp\{j2\pi(l-1)(l'-1)/N\}/\sqrt{N}$ for $l, l' = 1, \cdots, N$, and $\mathbf{s}_i$ is the ith data block of length N. In OFDM systems, elements $s_n[i]$ of the data block $\mathbf{s}_i$ are either a data symbol or zero which corresponds to a null carrier; in single-user MC-CDMA systems, $\mathbf{s}_i = \mathbf{a}b_i$ where $\mathbf{a}$ is the spreading code

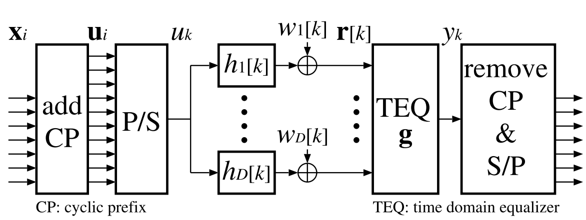

Fig. 1 SIMO block transmission model.

vector and b_i is a data symbol. Throughout the paper, we assume that the data symbols, $\{s_n[i] \neq 0\}$ or $\{b_i\}$, are uncorrelated with each other; then the assumption in (1) holds. For the later convenience, we define $\mathbf{\Phi} = E[\mathbf{x}_i\mathbf{x}_i^H]$ whose (n, n')th element is $\phi_{nn'}$.

In block transmission systems using CP, the last P elements of $\mathbf{x}_i$ are added to its beginning to form the ith transmitted block $\mathbf{u}_i$ whose nth element is $u_{iQ+n} = x_{(n+N-P)_N}[i]$ for $n = 0, \cdots, Q-1$ where $(m)_N$ is the residue of m modulo N and $Q = N + P$. Then, the received signal of the dth antenna can be expressed as

$$r_d(t) = \sum_{k=-\infty}^{\infty} u_k h_d(t - kT_s) + w_d(t) \tag{2}$$

for $d = 1, \cdots, D$, where $h_d(t)$ is the impulse response of the original channel, which includes transmitter and receiver filters as well as the physical channel, D is the number of antennae, $w_d(t)$ is a stationary spatially white Gaussian noise and uncorrelated with the information sequence, and $T_s = T_Q/Q$ where T_Q is the duration of the transmitted block with CP. We assume that $h_d(t)$ has finite support $[0, (M+1)T_s)$ where M is an integer and assumed to be $M \leq N$.

The received signals $r_d(t)$ are sampled at $t = k\Delta$ where $\Delta = T_s/O$ and the integer O is the over-sampling factor. The resulting discrete signal at the dth antenna can be written as

$$r_d[lO + k] = \sum_{m=0}^{M} h_d[mO + k]u_{l-m} + w_d[lO + k] \tag{3}$$

for $k = 0, \cdots, O-1$, where $r_d[k] = r_d(k\Delta), h_d[k] = h_d(k\Delta)$, and $w_d[k] = w_d(k\Delta)$. The sampled noise $w_d[k]$ is assumed to have the following correlation property:

$$E\left[w_d[k]\, w^*_{d'}[k']\right] = \begin{cases} \sigma_w^2 & d = d' \text{ and } k = k' \\ 0 & \text{otherwise} \end{cases} . \tag{4}$$

We will discuss the validity of the assumption later. In order to obtain a vector model, let us define the received sample vector of length LDO

$$\mathbf{r}[k] = \begin{bmatrix} \mathbf{r}_k \\ \mathbf{r}_{k-1} \\ \vdots \\ \mathbf{r}_{k-L+1} \end{bmatrix} = \mathbf{H}\mathbf{u}[k] + \mathbf{w}[k] \tag{5}$$

where $\mathbf{r}_k = [\bar{\mathbf{r}}_{kO}^T \cdots \bar{\mathbf{r}}_{(k+1)O-1}^T]^T$, $\bar{\mathbf{r}}_k = [r_1[k] \cdots r_D[k]]^T$, $\mathbf{u}[k] = [u_k \cdots u_{k-(L+M)+1}]^T$, $\mathbf{w}[k] = [\mathbf{w}_k^T \cdots \mathbf{w}_{k-L+1}^T]^T$, $\mathbf{w}_k = [\bar{\mathbf{w}}_{kO}^T \cdots \bar{\mathbf{w}}_{(k+1)O-1}^T]^T$, $\bar{\mathbf{w}}_k = [w_1[k] \cdots w_D[k]]^T$, and the $LDO \times (L+M)$ channel matrix $\mathbf{H}$ is

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}[0] \cdots \mathbf{h}[M] & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \ddots & & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{h}[0] \cdots \mathbf{h}[M] \end{bmatrix} \tag{6}$$

where $\mathbf{h}[i] = [\bar{\mathbf{h}}_{iO}^T \cdots \bar{\mathbf{h}}_{(i+1)O-1}^T]^T$, $\bar{\mathbf{h}}_k = [h_1[k] \cdots h_D[k]]^T$, and $M + 1$ is the length of the discrete channel impulse response. While intersymbol interference caused in the dispersive channel can be rejected due to CP if $M \leq P$, residual interference called interblock interference (IBI) degrades performance if $M > P$.

To cancel the IBI, a linear equalizer is employed. Let $\mathbf{g} = [g_0\, g_1\, \cdots\, g_{LDO-1}]^T$ be the parameter vector of an equalizer. The output of the equalizer is given by

$$y_k = \mathbf{g}^H \mathbf{r}[k] = \mathbf{c}^H \mathbf{u}[k] + \mathbf{g}^H \mathbf{w}[k] \tag{7}$$

where the superscript H represents the conjugate transpose of a matrix and

$$\mathbf{c} = \mathbf{H}^H \mathbf{g} = [c_0\, c_1\, \cdots\, c_{L+M-1}]^T. \tag{8}$$

The vector $\mathbf{c}$ represents the impulse response of the effective channel consisting of the original channel and equalizer. We call the equalizer $\mathbf{g}$ time-domain equalizer (TEQ) though the use of multiple antennae since the effective channel can be regarded as a time-domain system. Our goal is to find a vector $\mathbf{g}$ using only the channel output such that $\mathbf{c}$ becomes zero except consecutive $P + 1$ elements to cancel IBI.

3. Performance Limits of Conventional Method

3.1 MERRY Algorithm

The MERRY algorithm [14] is originally a stochastic gradient algorithm minimizing the cost function given by

$$J_\delta(\mathbf{g}) = E\left[|y_{iQ+P-1+\delta} - y_{iQ+Q-1+\delta}|^2\right] \tag{9}$$

where $\delta \in \{0, 1, \cdots, Q-1\}$ is a decision delay parameter and $E[\cdot]$ represents the ensemble average. To avoid the trivial solution $\mathbf{g} = \mathbf{0}$, the norm constraint $\|\mathbf{g}\| = 1$ is imposed. The cost function can be rewritten by $J_\delta(\mathbf{g}) = \mathbf{g}^H \mathbf{R}_\delta \mathbf{g}$ where $\mathbf{R}_\delta = E[\tilde{\mathbf{r}}[i]\tilde{\mathbf{r}}^H[i]]$ and $\tilde{\mathbf{r}}[i] = \mathbf{r}[iQ+P-1+\delta] - \mathbf{r}[iQ+Q-1+\delta]$. Then, the vector $\mathbf{g}$ minimizing the cost function under the norm constraint is an eigenvector corresponding to the minimum eigenvalue of $\mathbf{R}_\delta$. Hence, a batch algorithm is available, in which $\mathbf{R}_\delta$ is estimated by time-averaging and then eigendecomposition is performed. Throughout the paper, for simplicity, we assume that $\delta = 0$. In this case, due to the existence of CP, $\mathbf{R}_0$ becomes

$$\mathbf{R}_0 = \mathbf{H} \begin{bmatrix} \mathbf{0}_{P\times P} & \mathbf{0}_{P\times(L+M-P)} \\ \mathbf{0}_{(L+M-P)\times P} & \tilde{\mathbf{R}}_u \end{bmatrix} \mathbf{H}^H$$

where $\mathbf{0}_{m\times n}$ is the $m\times n$ zero matrix and $\tilde{\mathbf{R}}_u$ is the $(L+M-P)\times(L+M-P)$ matrix whose (l, l')th element is $\tilde{r}_{ll'} = E[(u_{iQ-l} - u_{iQ+N-l})(u_{iQ-l'} - u_{iQ+N-l'})^*]$ for $l, l' = 1, \cdots, L+M-P$.

3.2 Uncorrelated Signal Case

Let us consider the case where transmitted samples $x_n[i]$ are uncorrelated. In [17], a sufficient condition in order for the IBI component to be canceled perfectly by the MERRY algorithm has been derived. Here, we give a necessary and sufficient condition as follows:

Proposition 1: Assume that $\phi_{nn'} = 0$ for $n \neq n'$, and σ_x^2 for $n = n'$. Let $\mathbf{H}_{\text{ibi}}$ be the $LDO \times (L+M-P)$ matrix consisting of the last $L+M-P$ columns of $\mathbf{H}$. If and only if $\text{rank}(\mathbf{H}_{\text{ibi}}) < LDO$, $\mathbf{H}_{\text{ibi}}^H \mathbf{g} = \mathbf{0}$ by minimizing J_0.

Proof: Taking into account the assumption on the statistics of the transmitted samples, the cost function can be rewritten as

$$J_0(\mathbf{g}) = 2\sigma_x^2 \mathbf{g}^H \mathbf{H}_{\text{ibi}} \mathbf{H}_{\text{ibi}}^H \mathbf{g} + 2\sigma_w^2. \tag{10}$$

The first term in the right-hand side corresponds to the magnitude of $\mathbf{H}_{\text{ibi}}^H \mathbf{g} = \mathbf{c}_{\text{ibi}} = [c_P \cdots c_{L+M-1}]^T$. When $\text{rank}(\mathbf{H}_{\text{ibi}}) = LDO$, $\mathbf{H}_{\text{ibi}}\mathbf{H}_{\text{ibi}}^H$ becomes positive definite. Then, $J_0(\mathbf{g}) > 2\sigma_w^2$ for any $\mathbf{g} \neq \mathbf{0}$. This implies $\mathbf{c}_{\text{ibi}} \neq \mathbf{0}$ even if J_0 is minimized. On the other hand, when $\text{rank}(\mathbf{H}_{\text{ibi}}) < LDO$, there exists $\mathbf{g} \neq \mathbf{0}$ such that $\mathbf{H}_{\text{ibi}}^H \mathbf{g} = \mathbf{0}$ and then J_0 is minimized. This means that $\mathbf{c}_{\text{ibi}} = \mathbf{0}$, i.e. IBI can be canceled, by minimizing J_0. □

The condition $\text{rank}(\mathbf{H}_{\text{ibi}}) < LDO$ can be satisfied when $L+M-P < LDO$ which is also easily satisfied by increasing the number of antennae D or the over-sampling factor O.

As mentioned in Sect. 2, when the CP of length P is used, the impulse response length is allowed to be $P + 1$ to avoid IBI. However, the impulse response of the effective channel obtained by MERRY is of length P which is short by one.

Note that the MERRY algorithm does not ensure the remaining (desired) part of $\mathbf{c}$, i.e., $\mathbf{c}_{\text{des}} = [c_0\, \cdots\, c_{P-1}]^T$, is nonzero. A simple idea to prevent $\mathbf{c}_{\text{des}} = \mathbf{0}$ is to select $\mathbf{g}$ from the nullspace of $\mathbf{H}_{\text{ibi}}^H$ such that the TEQ output power is larger than σ_w^2 since $E[|y_k|^2] = \sigma_x^2 \|\mathbf{c}_{\text{des}}\|^2 + \sigma_w^2$.

3.3 Correlated Signal Case

Next, we consider the case where a transmitted sample $x_n[i]$ is correlated with another sample $x_{n'}[i]$ in the same block. In this case, the cost function can be written as

$$J_0(\mathbf{g}) = \mathbf{g}^H \mathbf{H}_{\text{ibi}} \tilde{\mathbf{R}}_u \mathbf{H}_{\text{ibi}}^H \mathbf{g} + 2\sigma_w^2. \tag{11}$$

When $Q \geq L+M$, the (l, l')th element of $\tilde{\mathbf{R}}_u$ can be expressed as

$$\tilde{r}_{ll'} = \phi_{N-l,N-l'} + \phi_{N-P-l,N-P-l'} \tag{12}$$

since $E[u_{iQ-l} u^*_{iQ+N-l'}] = 0$.

We can now give a sufficient condition in order for MERRY to cancel IBI when the transmitted samples within a block before the CP insertion are correlated.

Proposition 2: If $\mathbf{H}_{\text{ibi}}$ has full column rank and $\tilde{\mathbf{R}}_u$ has full rank, $\mathbf{c}_{\text{ibi}} = \mathbf{0}$ by minimizing J_0.

Proof: From the assumption, it follows that

$$\text{rank}(\mathbf{H}_{\text{ibi}} \tilde{\mathbf{R}}_u \mathbf{H}_{\text{ibi}}^H) = L + M - P < LDO. \tag{13}$$

By minimizing J_0, we obtain an eigenvector $\mathbf{g}$ such that $\mathbf{H}_{\text{ibi}} \tilde{\mathbf{R}}_u \mathbf{H}_{\text{ibi}}^H \mathbf{g} = \mathbf{0}$. Then, $\mathbf{c}_{\text{ibi}} = \mathbf{H}_{\text{ibi}}^H \mathbf{g} = \mathbf{0}$ since $\mathbf{H}_{\text{ibi}} \tilde{\mathbf{R}}_u$ has full column rank. □

On the other hand, when $\mathbf{H}_{\mathrm{ibi}}$ has full column rank but $\tilde{\mathbf{R}}_u$ does not have full rank, we obtain $\mathbf{g}$ by minimizing J_0 such that $\tilde{\mathbf{R}}_u\mathbf{H}_{\mathrm{ibi}}^H\mathbf{g} = \tilde{\mathbf{R}}_u\mathbf{c}_{\mathrm{ibi}} = \mathbf{0}$. Since $\tilde{\mathbf{R}}_u$ does not have full rank, the above system has a nontrivial solution $\mathbf{c}_{\mathrm{ibi}} \neq \mathbf{0}$. Thus, in this case, we might fail to cancel the interference.

It follows from a well-known result concerning blind identification [25] that $\mathbf{H}_{\mathrm{ibi}}$ has full column rank if $\sum_{m=0}^{M}\mathbf{h}[m]z^{-m} \neq \mathbf{0}$ for all z. Meanwhile, the rank of $\tilde{\mathbf{R}}_u$ depends on the statistical property of the transmitted signal. We now show that $\tilde{\mathbf{R}}_u$ can be rank-deficient in both OFDM with null carriers and MC-CDMA.

Proposition 3: Consider an OFDM system where $N_c < N$ carriers are used. Suppose $Q \geq L + M$. If $N_c < L + M - P$, $\tilde{\mathbf{R}}_u$ does not have full rank.

Proof: See Appendix A.
This suggests that MERRY might fail to shorten properly when the channel impulse response is long.

Proposition 4: Consider a single-user MC-CDMA system. Suppose that $Q \geq L+M$ and $L+M-P > 2$. Then, $\tilde{\mathbf{R}}_u$ does not have full rank.

Proof: See Appendix B.
Though we focus on a single-user system to show in a simple way that the performance of MERRY is limited, this result could be applied to a multiuser MC-CDMA downlink transmission as long as a the number of users is small. More consideration such as an extension to an uplink transmission is out of the scope of this paper.

4. Channel Shortening for Correlated Signals

Now we propose a new blind channel shortening method using the property in Eq. (1), i.e., a time-domain information block is uncorrelated with different blocks.

4.1 Principle

We assume that the block synchronization is perfect. Let $\tilde{\mathbf{R}}$ be a matrix which consists of the received signal vectors over time interval from $iQ + P$ to $iQ + P + J - 1$:

$$\tilde{\mathbf{R}} = \begin{bmatrix} \mathbf{r}^H[iQ+P] \\ \vdots \\ \mathbf{r}^H[iQ+P+J-1] \end{bmatrix}. \tag{14}$$

Denote $\tilde{\mathbf{U}}$ and $\tilde{\mathbf{W}}$ as the corresponding matrices consisting of the transmitted samples and noise vectors:

$$\tilde{\mathbf{U}} = \begin{bmatrix} \mathbf{u}^H[iQ+P] \\ \vdots \\ \mathbf{u}^H[iQ+P+J-1] \end{bmatrix}, \tag{15}$$

$$\tilde{\mathbf{W}} = \begin{bmatrix} \mathbf{w}^H[iQ+P] \\ \vdots \\ \mathbf{w}^H[iQ+P+J-1] \end{bmatrix}. \tag{16}$$

Then we have

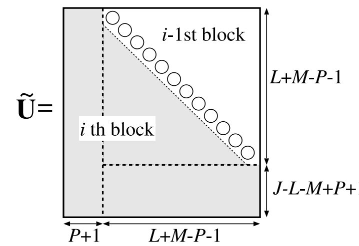

Fig. 2 Structure of matrix $\tilde{\mathbf{U}}$.

$$\tilde{\mathbf{R}} = \tilde{\mathbf{U}}\mathbf{H}^H + \tilde{\mathbf{W}}. \tag{17}$$

In Fig. 2, the structure of $\tilde{\mathbf{U}}$ is illustrated. The shaded part consists of the signals from the ith block and the white part consists of ones from the $i-1$st block. The diagonal elements of the top right square matrix, depicted by circles, are u_{iQ-1}.

Let us consider the correlation of $\tilde{\mathbf{R}}$ and a received sample $r_1[(iQ-1)O]$ defined by

$$\mathbf{R} = E\left[r_1[(iQ-1)O]\,\tilde{\mathbf{R}}\right]. \tag{18}$$

The received sample $r_1[(iQ-1)O]$ can be written as

$$r_1[(iQ-1)O] = \sum_{m=0}^{M} h_1[mO]u_{iQ-1-m} + w_1[(iQ-1)O] \tag{19}$$

Note that $r_1[(iQ-1)O]$ contains the transmitted samples of the $i-1$st block, not ith block and later. As shown in Fig. 2, the $i-1$st block components are not included in the first $P+1$ columns and the last $J-L-M+P+1$ rows of $\tilde{\mathbf{U}}$. Hence we may write

$$E[r_1[(iQ-1)O]\tilde{\mathbf{U}}] = \begin{bmatrix} \mathbf{0} & \mathbf{A} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \tag{20}$$
$$\qquad\qquad\qquad\qquad \underbrace{\quad}_{P+1}\ \underbrace{\quad}_{L+M-P-1}$$

where a square matrix $\mathbf{A}$ has the following property:

Lemma 5: When $J \geq L + M - P - 1$ and $\sum_{m=0}^{M} h_1[mO]\phi_{N-1,N-1-m} \neq 0$, $\mathbf{A}$ has full rank.

Proof: When $J \geq L + M - P - 1$, $\mathbf{A}$ is an $(L + M - P - 1) \times (L + M - P - 1)$ upper triangle matrix and its diagonal elements are $a_d = E[r_1[i(Q-1)O]u_{iQ-1}] = \sum_{m=0}^{M} h_1[mO]\phi_{N-1,N-1-m}$. Since $a_d \neq 0$ from the assumption, $\mathbf{A}$ has full rank. □
Then we have

$$\mathbf{R} = \begin{bmatrix} \mathbf{0} & \mathbf{A} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}\mathbf{H}^H + E[r_1[(iQ-1)O]\tilde{\mathbf{W}}]. \tag{21}$$

We can now establish the main channel shortenability result.

Theorem 6: Let $\tilde{\mathbf{H}}_{\mathrm{ibi}}$ be the $LDO \times (L+M-P-1)$ matrix obtained by deleting the first $P+1$ columns of $\mathbf{H}$. When $J \geq L+M-P-1, L < P+2, \mathrm{rank}(\tilde{\mathbf{H}}_{\mathrm{ibi}}) < LDO$, and $\sum_{m=0}^{M} h_1[mO]\phi_{N-1,N-1-m} \neq 0$, the length of the effective channel impulse response is shorter than or equal to $P+1$ by using $\mathbf{g}$ such that $\mathbf{Rg} = \mathbf{0}$.

Proof: The received sample $r_1[(iQ-1)O]$ includes a noise component $w_1[(iQ-1)O]$ which is also included in $\mathbf{r}_{iQ-1}$. If $L < P+2$, $\mathbf{r}_{iQ-1}$ is not included in $\tilde{\mathbf{R}}$ and thus the second term in the right-hand side of Eq. (21) becomes zero. Then we have $\mathbf{R} = \mathbf{A}\tilde{\mathbf{H}}_{\mathrm{ibi}}^H$. Since the matrix $\mathbf{A}$ has full rank according to Lemma 5,

$$\mathrm{rank}(\mathbf{R}) = \mathrm{rank}(\tilde{\mathbf{H}}_{\mathrm{ibi}}) < LDO. \tag{22}$$

The dimension of the nullspace of $\mathbf{R}$ is $LDO - \mathrm{rank}(\tilde{\mathbf{H}}_{\mathrm{ibi}}) > 0$. Thus, $\mathbf{g}$ can be chosen from the nullspace of $\mathbf{R}$, and then IBI becomes $\tilde{\mathbf{H}}_{\mathrm{ibi}}\mathbf{g} = [c_{P+1} \ \cdots \ c_{L+M-1}]^T = \mathbf{0}$. □

The received sample $r_1[(iQ-1)O]$ is not the only choice in our method. We can use the other received sample $r_d[(iQ-l)O+k], 1 \le l \le Q, k = 0, \cdots, O-1, d = 1, \cdots, D$, which contains the transmitted samples of the $i-1$st block, not ith block and later.

The condition $\mathrm{rank}(\tilde{\mathbf{H}}_{\mathrm{ibi}}) < LDO$ can be easily satisfied by increasing D or O so that $\tilde{\mathbf{H}}_{\mathrm{ibi}}$ can be tall, i.e., $L+M-P-1 < LDO$. The condition $J \ge L+M-P-1$ can be also satisfied by a moderately large J. These two conditions require a rough estimate of M. It is not difficult to estimate M roughly by using AIC or MDL which are popular tools in blind receivers [26]. Though, if $L \ge P+2$, the noise component remains, it might not be serious as demonstrated in our simulation since the contribution of the noise on $\mathbf{R}$ is very little.

4.2 Application to OFDM Systems

The time-domain block in OFDM systems can be written as $\mathbf{x}_i = \mathbf{F}\mathbf{s}_i$ where $\mathbf{s}_i = [s_0[i] \ \cdots \ s_{N-1}[i]]^T$ is the data block vector and $\mathbf{F}$ is the IFFT matrix. If all N carriers are used, the autocorrelation of the time-domain block becomes $\mathbf{\Phi} = E[\mathbf{x}_i\mathbf{x}_i^H] = \mathbf{F}E[\mathbf{s}_i\mathbf{s}_i^H]\mathbf{F}^H = \sigma_s^2\mathbf{I}$, that is, $\phi_{nn'} = \sigma_s^2$ for $n = n'$ and 0 otherwise. Then, the diagonal element of $\mathbf{A}$ becomes

$$a_d = \sum_{m=0}^{M} h_1[mO]\phi_{N-1,N-1-m} = h_1[0]\sigma_s^2. \tag{23}$$

Hence if $h_1[0] \neq 0$ the proposed method can work.

Next, consider an OFDM system where $N_c < N$ carriers are used. Without loss of generality, we assume that null carriers are placed at the tail of the data block, $\mathbf{s}_i = [s_0[i] \ \cdots \ s_{N_c-1}[i] \ 0 \cdots \ 0]$. Then we have $\mathbf{\Phi} = E[\mathbf{x}_i\mathbf{x}_i^H] = \sigma_s^2\tilde{\mathbf{F}}\tilde{\mathbf{F}}^H$ where $\tilde{\mathbf{F}}$ is composed of the first N_c columns of $\mathbf{F}$. Then the diagonal element of $\mathbf{A}$ is

$$a_d = \sigma_s^2 \sum_{m=0}^{M} h_1[mO] \left(\frac{1}{N} \sum_{l=0}^{N_c-1} e^{-j\frac{2\pi}{N}lm} \right). \tag{24}$$

Suppose that the channel coefficients $\{h_d[k]\}$ can be modeled as complex Gaussian variables, then the probability that $a_d = 0$ is zero. In this sense, the proposed method can be applied to OFDM systems.

4.3 Application to MC-CDMA System

Let us consider a single-user MC-CDMA system where the time-domain block is given by $\mathbf{x}_i = \mathbf{F}\mathbf{s}_i$ where $\mathbf{s}_i = \mathbf{a}b_i$, $\mathbf{a}$ is the spreading code of length N, and b_i is the ith data symbol. Autocorrelation matrix of $\mathbf{x}_i$ can be written as $\mathbf{\Phi} = E[\mathbf{x}_i\mathbf{x}_i^H] = \sigma_b^2\mathbf{f}\mathbf{f}^H$ where $\mathbf{f} = \mathbf{F}\mathbf{a}$ and $\sigma_b^2 = E[|b_i|^2]$. Then, the diagonal elements of $\mathbf{A}$ is

$$a_d = \sigma_b^2 f_{N-1} \sum_{m=0}^{M} h_1[mO] f_{N-1-m} \tag{25}$$

where $f_l, l = 0, \cdots, N-1$, is the lth element of $\mathbf{f}$. If $\{h_d[k]\}$ can be modeled as complex Gaussian variables, the probability that $a_d = 0$ is zero unless $f_{N-1} = 0$. In this sense, the proposed method works in MC-CDMA.

4.4 Algorithm

A batch processing algorithm based on the above principle is detailed next. The ensemble average in (18) is evaluated by the time average over B blocks.

Step1) Construct matrices $\tilde{\mathbf{R}}[i], i = 0, \cdots, B-1$ from the received signal vector.

$$\tilde{\mathbf{R}}[i] = \begin{bmatrix} \mathbf{r}^H[iQ+P] \\ \vdots \\ \mathbf{r}^H[iQ+P+J-1] \end{bmatrix}.$$

Step2) Obtain the estimate of $\mathbf{R}$ by time-averaging.

$$\hat{\mathbf{R}} = \frac{1}{B} \sum_{i=0}^{B-1} r_1[i(Q-1)O]\tilde{\mathbf{R}}[i].$$

Step3) Compute the singular value decomposition of $\hat{\mathbf{R}}$, $\hat{\mathbf{R}} = \mathbf{Q}_1\mathbf{\Sigma}\mathbf{Q}_2^H$, and the last column vector of $\mathbf{Q}_2^H$ is set to $\mathbf{g}$.

5. Simulation

In this section, we show computer simulation results illustrating the performance of the proposed algorithm in both OFDM and MC-CDMA systems. The number of receive antennae was $D = 2$. Channel coefficients $\{h_d[i]\}$ were independent zero-mean complex Gaussian random variables with unit variance[†]. The information bits were mapped onto QPSK symbols. The received SNR per antenna was defined by

$$\mathrm{SNR} = \frac{1}{QDO\sigma_w^2} \sum_{k=0}^{QO-1} \sum_{d=1}^{D} E[|\tilde{r}_d[k]|^2] \tag{26}$$

where $\tilde{r}_d[k]$ is the noiseless received sample. Unless otherwise stated, we set SNR=40 [dB], $N = 16$, $P = 4$, $M = 8$,

[†]Independent channel coefficients are useful to verify if the proposed method works. Even if the coefficients are not independent, which may arise in certain wireless systems, the proposed method works as long as the conditions presented in Theorem 6 are satisfied.

$L = 8$, $O = 1$, $J = 12$, and $B = 50,000$. Though the FFT size N was set to small to save computation time, the obtained results are enough to validate our theoretical results. The SNR is rather high to show the superiority of the proposed algorithm clearly. The performance measure was SINR defined as [18]:

$$\mathrm{SINR} = \frac{E\left[|\mathbf{c}_{\delta^*}^H \mathbf{u}_{\delta^*}[k]|^2\right]}{E\left[|y_k - \mathbf{c}_{\delta^*}^H \mathbf{u}_{\delta^*}[k]|^2\right]} \tag{27}$$

where $\mathbf{c}_\delta = [c_\delta \cdots c_{\delta+P}]^T$, $\mathbf{u}_\delta[k] = [u_{k-\delta} \cdots u_{k-\delta-P}]^T$ and $\delta^* = \arg\max_\delta \|\mathbf{c}_\delta\|^2$. Thus $\mathbf{c}_{\delta^*}$ represents the signal component consisting of the consecutive $P+1$ samples which has the largest contribution to the output power. Both SNR in (26) and SINR in (27) can be calculated if $\mathbf{g}, \mathbf{H}$ and the statistics of $\{u_k\}$ and $\{w_d[k]\}$ are available. SINR obtained by (27) was averaged over 100 different trials. The TEQ output SINR is an appropriate measure to evaluate the IBI rejection performance of blind TEQ since it reflects residual IBI.

5.1 OFDM

First we consider an OFDM system where the first N_c carriers are used and the remaining $N - N_c$ carriers are unused. Unless otherwise stated, we set $N_c = 10$. In Fig. 3, the example of the impulse response of original channels from the transmitter to receive antenna 1 and 2, and those of the shortened effective channels obtained by the MERRY algorithm and proposed algorithm are shown. In this case, $\tilde{\mathbf{R}}_u$ does not have full column rank since $N_c < L + M - P = 12$. As is seen, while MERRY fails to shorten the channel, the proposed algorithm can shorten the channel to $P + 1$ successfully. Figure 4 shows average SINR characteristics. The receiver without a TEQ has poor performance especially at high SNR due to residual IBI. The proposed algorithm can achieve a higher SINR than MERRY.

The average SINR performance of both the proposed and MERRY algorithm as a function of N_c is shown in Fig. 5. The performance of MERRY is poor when $N_c < L + M - P = 12$ due to the rank-deficiency. Though when $N_c \geq 2$ the performance of the proposed algorithm is satisfactory, it becomes worse as N_c increases. This is due to the inaccuracy of the estimated $\mathbf{R}$ by an insufficient number of blocks. More blocks are required as N_c increases.

Figure 6 displays the average SINR varied over B, the number of blocks used to estimate the matrix $\mathbf{R}$, when SNR is 20 dB. It can be observed that the performance of MERRY is poor and that of the proposed algorithm becomes better as B increases. This result suggests that the proposed algorithm requires a large number of blocks to achieve excellent performance.

In Fig. 7, the average SINR of the proposed algorithm as a function of TEQ taps per antenna L is shown and that of MERRY is also shown. The performance of MERRY is poor when $L > 6$ and $L < 4$ since $\tilde{\mathbf{R}}_u$ does not have full column rank when $L > N_c - M + P = 6$ and $\mathbf{H}_{\mathrm{ibi}}$ does not have full column rank when $L < M - P = 4$. It can be observed that the performance of the proposed algorithm is satisfactory when $L \geq 4$, which ensures $\mathrm{rank}(\tilde{\mathbf{H}}_{\mathrm{ibi}}) < LDO = 8$.

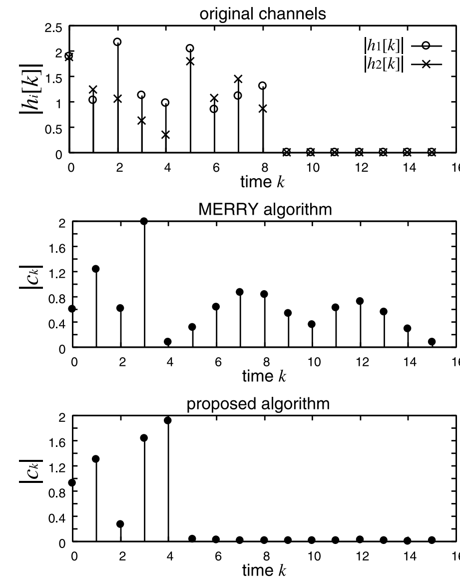

Fig. 3 Example of impulse response of original channel and shortened effective channels in OFDM system with null carriers.

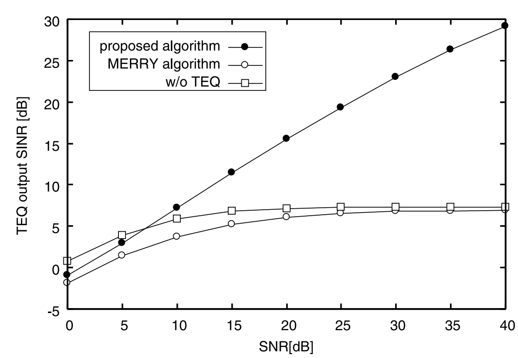

Fig. 4 SINR characteristics of various methods in OFDM system with null carriers.

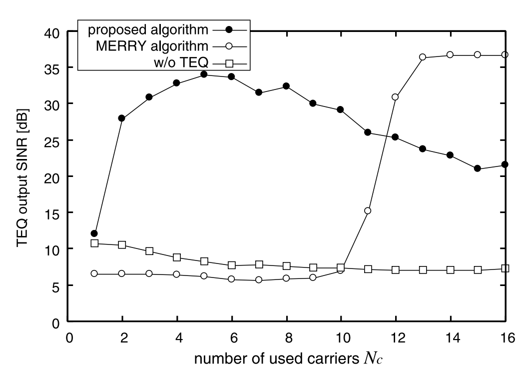

Fig. 5 SINR versus number of subcarriers N_c.

5.2 MC-CDMA

Next, we show the results of a single-user MC-CDMA sys-

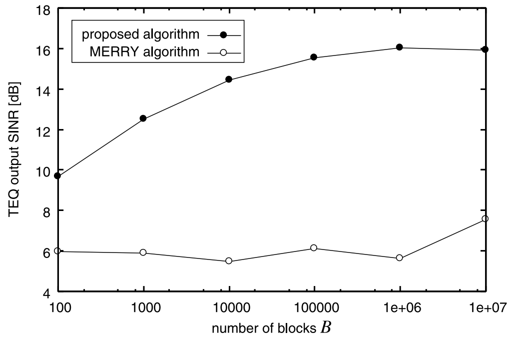

Fig. 6 SINR versus number of blocks B.

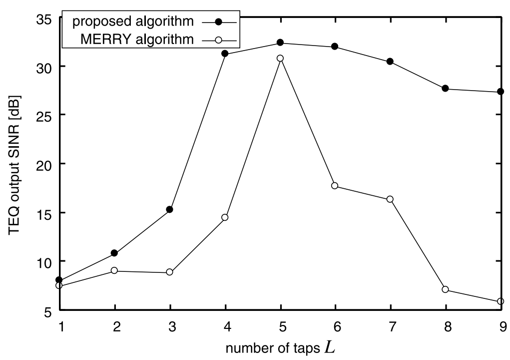

Fig. 7 SINR versus number of taps per antenna L.

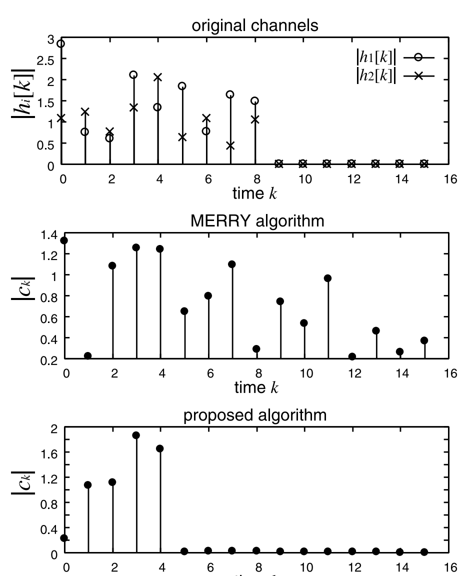

Fig. 8 Example of impulse response of original channel and shortened effective channels in MC-CDMA system.

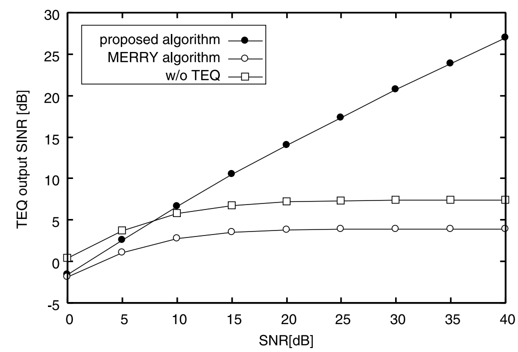

Fig. 9 SINR characteristics of various methods in MC-CDMA system.

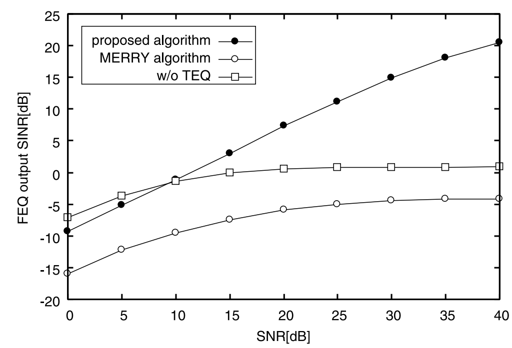

Fig. 10 FEQ output SINR characteristics of various methods in OFDM system with null carriers.

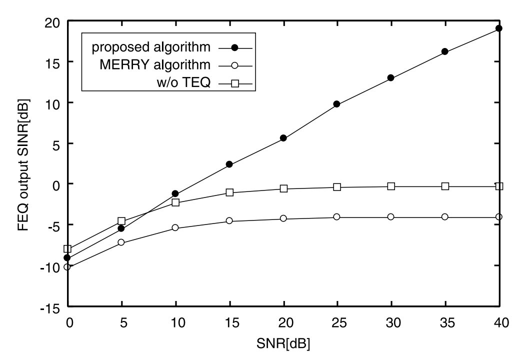

Fig. 11 FEQ output SINR characteristics of various methods in MC-CDMA system.

tem. The element of the spreading code **a** takes a value $\{+1, -1\}$ randomly. The spreading factor is $N = 16$. Average SINR was obtained by averaging over 100 different channels and codes. The impulse responses and SINR performance are shown in Figs. 8 and 9, respectively. As in the case of OFDM with null carriers, while MERRY does not work, the proposed algorithm can shorten the channel within $P + 1$ and achieve excellent SINR performance.

5.3 FEQ Output SINR

Finally, we consider a total performance of a receiver including TEQ and FEQ (frequency-domain equalizer). To obtain an FEQ, the effective channel impulse response is required. One approach might be blind channel estimation [27]. However, if we use these methods in conjunction with our TEQ, the performance of the TEQ cannot be evaluated clearly due to channel estimation error. Thus, we used the ideal channel impulse response. The FEQ output SINRs in OFDM with null carriers and MC-CDMA system are shown in Figs. 10 and 11, respectively, where a zero-forcing FEQ was employed. The results show that the performance of the proposed method is still better than MERRY.

6. Conclusion

In this paper we have considered blind channel shortening for correlated signals. The channel shortening capabilities of the conventional MERRY algorithm are limited when it is applied to both an OFDM with null carriers and an MC-CDMA system. It is shown that the proposed shortening method using the correlation property of the transmitted samples can be applied to both an OFDM and an MC-CDMA system under certain conditions. Simulation results verify the theoretical expectations and show the superiority of the proposed algorithms over a conventional one.

The assumption on the noise statistics in (4) is not always valid due to the effect of receive filters when $O > 1$. If the statistics of the noise can be obtained, the contribution by noise to J_δ or $\mathbf{R}$ can be estimated. Then, we obtain the same results as Propositions 1 and 2 and Theorem 6 by subtracting the noise contribution from J_δ or $\mathbf{R}$, respectively.

As shown in the simulation results, the proposed algorithm requires a large number of data blocks to achieve a satisfactory level of performance. To overcome this disadvantage is a challenging problem. Moreover, the proposed algorithm requires synchronization of the transmitted block. A simple blind synchronization method in [17] could be useful. A development of a precise synchronization method is an important issue.

References

[1] Z. Wang and G.B. Giannakis, "Wireless multicarrier communications," IEEE Signal Process. Mag., vol.17, no.3, pp.29–48, May 2000.

[2] S. Hara and R. Prasad, "Overview of multicarrier CDMA," IEEE Commun. Mag., vol.35, no.12, pp.126–133, Dec. 1997.

[3] D. Falconer, S.L. Ariyavisitakul, A. Benyamin-Seeyar, and B. Eidson, "Frequency domain equalization for single-carrier broadband wireless systems," IEEE Commun. Mag., vol.40, no.4, pp.58–66, April 2002.

[4] IEEE Std 802.11a, "Wireless LAN medium access control (MAC) and physical layer specifications: High speed physical layer in the 5 GHz band," Sept. 1999.

[5] P.J. Melsa, R.C. Younce, and C.E. Rohrs, "Impulse response shortening for discrete multitone transceivers," IEEE Trans. Commun., vol.44, no.12, pp.1662–1672, Dec. 1986.

[6] D. Kim and G.L. Stuber, "Residual ISI cancellation for OFDM with applications to HDTV broadcasting," IEEE J. Sel. Areas Commun., vol.50, no.12, pp.2136–2148, Dec. 2002.

[7] P. Alexander, D. Haley, and A. Grant, "Outdoor mobile broadband access with 802.11," IEEE Commun. Mag., vol.45, no.11, pp.108–114, Nov. 2007.

[8] A. Stamoulis, G.B. Giannakis, and A. Scaglione, "Block FIR decision-feedback equalizers for filterbank precoded transmissions with blind channel estimation capabilities," IEEE Trans. Commun., vol.16, no.8, pp.1590–1599, Oct. 1998.

[9] S. Hori, N. Kikuma, and N. Inagaki, "MMSE adaptive array utilizing guard interval in the OFDM systems," Electron. Commun. Jpn. 1, Commun., vol.86, no.10, pp.1–9, April 2003.

[10] K. Hayashi, T. Kojima, and H. Sakai, "An adaptive antenna array for single carrier modulation with cyclic prefix," Electron. Commun. Jpn. 1, Commun., vol.88, no.9, pp.38–48, April 2005.

[11] K.V. Acker, G. Leus, M. Moonen, O.V.D. Wiel, and T. Pollet, "Per tone equalization for DMT-based systems," IEEE Trans. Commun., vol.49, no.1, pp.109–119, Jan. 2001.

[12] R.K. Martin, K. Vanbleu, M. Ding, G. Ysebaert, M. Milosevic, B.L. Evans, M. Moonen, and C.R. Johonson, Jr., "Unification and evaluation of equalization structures and design algorithms for discrete multitone modulation systems," IEEE Trans. Signal Process., vol.53, no.10, pp.3880–3894, Oct. 2004.

[13] R.K. Martin and C.R. Johnson, Jr., "Adaptive equalization: Transitioning from single-carrier to multicarrier systems," IEEE Signal Process. Mag., vol.22, no.6, pp.108–122, Nov. 2005.

[14] R.K. Martin, J. Balakrishnan, W.A. Sethares, and C.R. Johnson, Jr., "A blind, adaptive TEQ for multicarrier receivers," IEEE Signal Process. Lett., vol.9, no.11, pp.341–343, Nov. 2002.

[15] R.K. Martin, J.M. Walsh, and C.R. Johnson, Jr., "Low-complexity MIMO blind, adaptive channel shortening," IEEE Trans. Signal Process., vol.53, no.4, pp.1324–1334, April 2005.

[16] R.K. Martin, "Fast-converging blind adaptive channel-shortening and frequency-domain equalization," IEEE Trans. Signal Process., vol.55, no.1, pp.102–110, Jan. 2007.

[17] H. Kameyama, T. Miyajima, and Z. Ding, "Perfect blind channel-shortening in multicarrier systems," IEEE Trans. Circuits Syst. I, vol.55, no.3, pp.851–860, April 2008.

[18] T. Miyajima and Z. Ding, "Second-order statistical approaches to channel-shortening in multicarrier systems," IEEE Trans. Signal Process., vol.52, no.11, pp.3253–3264, Nov. 2004.

[19] H. Gazzah and S. McLaughlin, "An SOS-based blind channel shortening algorithm," Proc. IEEE ICASSP, vol.4, Toulouse, May 2006.

[20] D. Darsena and F. Verde, "Minimum-mean-output-energy blind adaptive channel shortening for multicarrier SIMO transceivers," IEEE Trans. Signal Process., vol.55, no.10, pp.5755–5771, Oct. 2007.

[21] J. Balakrishnan, R.K. Martin, and C.R. Johnson, Jr., "Blind, adaptive channel shortening by sum-squared auto-correlation minimization(SAM)," IEEE Trans. Signal Process., vol.51, no.12, pp.3086–3093, Dec. 2003.

[22] R. Nawaz and J.A. Chambers, "Blind adaptive channel shortening by single lag autocorrelation minimisation," Electron. Lett., vol.40, no.25, pp.1609–1611, Dec. 2004.

[23] R. Romano and S. Barbarossa, "Non-data aided adaptive channel shortening for efficient multi-carrier systems," Proc. IEEE ICASSP, pp.233–236, Hong Kong, April 2003.

[24] IEEE Std. 802.16e-2005, "Air interface for fixed and mobile broadband wireless access systems," Feb. 2006.

[25] Z. Ding and Y. Li, Blind equalization and identification, Marcel Dekker, 2001.

[26] M. Wax and T. Kailath, "Detection of signals by information theoretic criteria," IEEE Trans. Acoust. Speech Signal Process., vol.ASSP-33, no.2, pp.387–392, April 1985.

[27] C. Li and S. Roy, "Subspace-based blind channel estimation for OFDM by exploiting virtual carriers," IEEE Trans. Wireless Commun., vol.2, no.1, pp.141–150, Jan. 2003.

Appendix A: Proof of Proposition 3

When $Q \geq L + M$, from Eq. (12), $\tilde{\mathbf{R}}_u = E\Big[\mathbf{u}_{Q-1}[i-1]\mathbf{u}_{Q-1}^H[i-1]\Big] + E\Big[\mathbf{u}_{N-1}[i]\mathbf{u}_{N-1}^H[i]\Big]$ where $\mathbf{u}_k[i] = [u_{iQ+k} \ \cdots \ u_{iQ+k-(L+M-P)+1}]$. Let us define two matrices $\mathbf{F}_1^H = [\mathbf{f}_N^H \ \cdots \ \mathbf{f}_{N-(L+M-P)+1}^H]$ and $\mathbf{F}_2^H = [\mathbf{f}_{N-P}^H \ \cdots \ \mathbf{f}_1^H \ \mathbf{f}_N^H \ \cdots \ \mathbf{f}_{N-(L+M-N)+1}^H]$ where $\mathbf{f}_l, l = 1, \cdots, N$, is the lth row of $\mathbf{F}$. Using them, we have $\mathbf{u}_{Q-1}[i-1] = \mathbf{F}_1\mathbf{s}_{i-1}$ and $\mathbf{u}_{N-1}[i] = \mathbf{F}_2\mathbf{s}_i$. Then, we have

$$\tilde{\mathbf{R}}_u = \mathbf{F}_1\mathbf{R}_s\mathbf{F}_1^H + \mathbf{F}_2\mathbf{R}_s\mathbf{F}_2^H \tag{A· 1}$$

where $\mathbf{R}_s = E\left[\mathbf{s}_i\mathbf{s}_i^H\right]$ is a diagonal matrix. In OFDM with null carriers, the lth diagonal element of $\mathbf{R}_s$ is σ_s^2 for $l \in C$ and 0 otherwise where σ_s^2 is the variance of $\{s_k[i] \neq 0\}$ and C is the set of the indexes of carriers used. Since $\tilde{\mathbf{R}}_u$ can be rewritten as $\tilde{\mathbf{R}}_u = [(\mathbf{F}_1\mathbf{R}_s)\ (\mathbf{F}_2\mathbf{R}_s)][(\mathbf{F}_1\mathbf{R}_s)\ (\mathbf{F}_2\mathbf{R}_s)]^H/\sigma_s^2$, $\mathrm{rank}(\tilde{\mathbf{R}}_u) = \mathrm{rank}([(\mathbf{F}_1\mathbf{R}_s)\ (\mathbf{F}_2\mathbf{R}_s)])$. The lth column of $\mathbf{F}_1\mathbf{R}_s$ is the lth column of $\mathbf{F}_1$ multiplied by σ_s^2 for $l \in C$ and a zero vector otherwise. The rank of $\mathbf{F}_1\mathbf{R}_s$ is equal to that of a matrix composed of the lth columns of $\mathbf{F}_1$ for $l \in C$. This matrix is an $(L + M - P) \times N_c$ Vandermonde matrix whose (n, m)th element is $\left(e^{-j\frac{2\pi}{N}i_m}\right)^{n-1}$, where i_m is the index of the mth used carrier, and thus has full column rank if $L + M - P \geq N_c$. Thus, the rank of $\mathbf{F}_1\mathbf{R}_s$ is also N_c. Since the ith column of $\mathbf{F}_1$ and that of $\mathbf{F}_2$ are linearly dependent, $\mathrm{rank}([(\mathbf{F}_1\mathbf{R}_s)\ (\mathbf{F}_2^H\mathbf{R}_s)]) = \mathrm{rank}(\mathbf{F}_1^H\mathbf{R}_s)$. Thus, if $N_c < L + M - P$, $\tilde{\mathbf{R}}_u$ whose rank is N_c is rank-deficient. □

Appendix B: Proof of Proposition 4

Define $\sigma_b^2 = E[|b_i|^2]$. Then we have $\mathbf{R}_s = \sigma_b^2\mathbf{a}\mathbf{a}^H$. From Eq. (A· 1), $\mathrm{rank}(\tilde{\mathbf{R}}_u) \leq \mathrm{rank}(\mathbf{F}_1\mathbf{a}\mathbf{a}^H\mathbf{F}_1^H) + \mathrm{rank}(\mathbf{F}_2\mathbf{a}\mathbf{a}^H\mathbf{F}_2^H)$. Since $\mathrm{rank}(\mathbf{F}_l\mathbf{a}\mathbf{a}^H\mathbf{F}_l^H) = 1$ for $l = 1, 2$, $\tilde{\mathbf{R}}_u$ is rank-deficient if $L + M - P > 2$. □

Teruyuki Miyajima received the B.Eng., M.Eng., and Ph.D. degrees in electrical engineering from Saitama University, Saitama, Japan, in 1989, 1991, and 1994, respectively. In 1994, he joined Ibaraki University, Hitachi, Japan, where he is currently an associate professor in the Department of Electrical and Electronic Engineering. His current interests are in statistical signal processing for communications. Dr. Miyajima is a member of IEEE.

Yoshihisa Watanabe received his B.Eng. and M.Eng. degrees in systems engineering from Ibaraki University, Hitachi, Japan, in 2006 and 2008, respectively. In 2008, he joined Mitsubishi Electric Corporation. His research interests include signal processing for communications.