PAPER

Blind Adaptive Beamformer for Cyclostationary Sources with Application to CDMA Systems

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SUMMARY In this paper, a simple blind algorithm for a beamforming antenna is proposed. This algorithm exploits the property of cyclostationary signals whose cyclic autocorrelation function depends on delay as well as frequency. The cost function is the mean square error between the delay product of the beamformer output and a complex exponential. Exploiting the delay greatly reduces the possibility of capturing undesired signals. Through analysis of the minima of the non-quadratic cost function, conditions to extract a single signal are derived. Application of this algorithm to code-division multiple-access systems is considered, and it is shown through simulation that the desired signal can be extracted by appropriately choosing the delay as well as the frequency.

key words: *cyclostationarity, DS/CDMA, adaptive array antenna, blind algorithm, higher-order statistics*

1. Introduction

Adaptive beamforming is an attractive technique for signal separation in wireless multiuser communications. There is especially a strong and practical need for blind adaptive beamforming that does not require use of a training sequence and thus avoids reducing the effective data rate. An effective and popular blind beamforming algorithm is the constant modulus algorithm (CMA) [1] which extracts a signal by restoring the constant modulus property disturbed by interference from other signals. The CMA beamformer extracts a single signal if the signal satisfies a certain condition regardless of whether the signal is desired or not [2]. This signal choice ambiguity, known as the capture problem, cannot be avoided as long as only the constant modulus property is used because this is a common property of communication signals. To resolve this ambiguity, additional signal information must be used.

Most communication signals are characterized by cyclostationarity, which depends on the modulation scheme, carrier frequency, data rate and so on [3]. The cyclostationarity can be exploited as additional information to extract the desired signal. The spectral self-coherence restoral (SCORE) algorithm [4] exploits the second-order cyclostationarity in which a signal is correlated with a frequency-shifted version of itself. The least-squares SCORE algorithm is simple to implement and works well as long as one can find a frequency parameter that ensures only the desired signal exhibits cyclostationarity. If several signals exhibit cyclostationarity at the given frequency, the least-squares SCORE algorithm cannot extract only the desired signal and tends to extract a mixture of the signals. Such a situation arises in a direct-sequence code-division multiple-access (DS/CDMA) system [5] where all users employ the same modulation scheme, carrier frequency, symbol rate, chip rate, and chip waveform, with the only difference among users being the spreading codes. More powerful SCORE variations such as phase SCORE [4] have an extremely high computational cost.

In [6], Castedo and Figueiras-Vidal proposed a blind algorithm, referred to as the CF algorithm, which exploits higher-order statistics regarding cyclostationary signals. The non-quadratic cost function is the mean square error between the square of the beamformer output and a complex exponential. The CF algorithm can extract the desired signal, though, if only the desired signal is cyclostationary with respect to the given frequency. Even if there are several signals which exhibit cyclostationarity for a given frequency, the CF algorithm tends to extract one of these signals, rather than a linear combination of them. However, signal choice ambiguity regarding these cyclostationary signals is still a problem.

In this paper, we propose a modified CF algorithm that exploits the property of cyclostationary signals whose cyclic autocorrelation function depends on delay as well as frequency shift. In the new cost function, the square of the beamformer output in the CF cost function is replaced with the delay product. The proposed algorithm can be implemented through a simple stochastic gradient algorithm. Exploiting the delay reduces the capture problem. We show that the proposed algorithm can extract only the desired signal successfully in CDMA communications where the SCORE and CF algorithms fail.

There are currently several blind techniques exploiting cyclostationarity, in which the delay as well as the frequency shift is taken into account [7]–[9]. In [7] and [8], blind signal separation was considered, thus the signal choice ambiguity inherent to blind signal separation still exists. The cost function presented in [9] is the mean squared error between the sum of weighted complex exponentials and a delay product of the beamformer output. The delay product is the product of the output and the delayed conjugate output, whereas the delay product without the conjugate is used in the algorithm presented in this paper. The properties of its minima have not been analyzed, though, so it is not clear whether the algorithm can extract only the desired signal when several sig-

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nals are cyclostationary with respect to the same frequency.

This paper is organized as follows. Section 2 presents the problem formulation of blind beamforming and briefly introduces cyclostationarity. Section 3 introduces a new algorithm and analyzes the properties of the stationary points of the cost function. In Sect. 4, application of the proposed algorithm to CDMA systems is discussed. Section 5 shows simulation results to support our analytical results.

2. Cyclostationarity of Communication Signals

2.1 Communication Model

Let us consider a multiuser communication system where there are K independent signals. The signal vector received by an array of N antennae, denoted by the $N \times 1$ vector $\mathbf{x}(t)$, is described by the following model

$$\mathbf{x}(t) = \sum_{k=1}^{K} s_k(t)\mathbf{a}(\theta_k) + \mathbf{n}(t), \tag{1}$$

where $s_k(t)$ is the signal arriving from the direction θ_k, $\mathbf{a}(\theta_k)$ is the steering vector associated with θ_k, and $\mathbf{n}(t) = [n_1(t), \cdots, n_N(t)]^T$ is a noise vector. We define an $N \times K$ mixing matrix formed from K steering vectors as

$$\mathbf{A} \triangleq [\mathbf{a}(\theta_1),\ \mathbf{a}(\theta_2),\ \cdots\ , \mathbf{a}(\theta_K)\ ]\,. \tag{2}$$

The beamformer output is given by

$$y(t) = \mathbf{w}^H \mathbf{x}(t), \tag{3}$$

where $\mathbf{w} = [w_1, w_2, \cdots, w_N]^T$ is the array weight vector and $(\cdot)^H$ represents the Hermitian transpose of a matrix. In the following, the first signal is treated as the desired signal. The purpose of the adaptive beamformer is to extract the desired signal $s_1(t)$ from the received signal $\mathbf{x}(t)$ by adjusting the weight vector $\mathbf{w}$.

2.2 Cyclostationarity

In communication applications, the source signals can be modeled as cyclostationary signals. This cyclostationarity has been exploited in many signal processing tasks such as signal detection and parameter estimation [3]. The second-order cyclostationarity is characterized by the cyclic autocorrelation function and the conjugate cyclic autocorrelation function of $s_k(t)$ defined by

$$\tilde{m}^{\alpha}_{2k}(\tau) \triangleq \left\langle s_k(t)s_k^*(t-\tau)e^{-i2\pi\alpha t}\right\rangle, \tag{4}$$

$$m^{\alpha}_{2k}(\tau) \triangleq \left\langle s_k(t)s_k(t-\tau)e^{-i2\pi\alpha t}\right\rangle, \tag{5}$$

where $(\cdot)^*$ represents the conjugate and the operation $\langle\cdot\rangle$ is the time-averaging operation:

$$\langle\cdot\rangle \triangleq \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} (\cdot)dt. \tag{6}$$

The frequency shift parameter α is referred to as the cycle frequency. A signal $s_k(t)$ is said to be cyclostationary if the cyclic autocorrelation function or conjugate cyclic autocorrelation function is nonzero at some frequency shift α and some delay τ.

In this paper, we consider linear digital modulated signals whose analytical representation is given by

$$s_k(t) = \left(A_k \sum_n I_{kn} a_k(t - nT_k)\right) e^{i2\pi f_{ck} t}, \tag{7}$$

where A_k is the signal amplitude, f_{ck} is the carrier frequency (which is assumed to be nonzero), I_{kn} is a data sequence with rate $1/T_k$, and $a_k(t)$ is the pulse waveform. For example, binary phase shift keying (BPSK) signals, $I_{kn} \in \{\pm 1\}$, with rectangular pulse, $a_k(t) = 1$ for $t \in [0, T_k]$ and 0 for $t \notin [0, T_k]$, have nonzero conjugate cyclic autocorrelations at $\alpha = 2f_{ck} \pm\ l/T_k, l = 0, 1, \cdots$, for nonzero delays τ.

2.3 Conventional Algorithms

The first blind beamforming algorithm to use cyclostationarity was the SCORE algorithm [4]. The cost function of the SCORE algorithm is defined by

$$J_s \triangleq \left\langle |y(t) - r(t)|^2 \right\rangle, \tag{8}$$

where $r(t)$ is the reference signal given by $r(t) = \mathbf{c}^H\mathbf{x}(t - \tau)e^{i2\pi\alpha t}$ and $\mathbf{c}$ is the control vector which should be chosen so that it is not orthogonal to $\mathbf{a}(\theta_1)$. The solution which minimizes the cost function is given by

$$\mathbf{w}_s = \hat{\mathbf{R}}_{xx}^{-1}\hat{\mathbf{R}}_{xr}, \tag{9}$$

where $\hat{\mathbf{R}}_{xx}$ and $\hat{\mathbf{R}}_{xr}$ are, respectively, the sample autocorrelation matrix and cross-correlation vector. If only the desired signal $s_1(t)$ has nonzero cyclic autocorrelation $\tilde{m}^{\alpha}_{21}(\tau) \neq 0$ for given α and τ, the vector $\hat{\mathbf{R}}_{xr}$ is proportional to $\mathbf{a}(\theta_1)$. This means that the weight vector $\mathbf{w}_s$ converges to the maximum SINR solution. On the other hand, if some signals have nonzero cyclic autocorrelation for the given α and τ, the SCORE algorithm extracts a mixture of these signals.

An important blind beamforming algorithm based on higher-order cyclostationary signal statistics is the CF algorithm [6]. The cost function of the CF algorithm is defined by

$$J_c \triangleq \left\langle |e^{i2\pi\alpha t} - y^2(t)|^2 \right\rangle. \tag{10}$$

The corresponding stochastic gradient algorithm is given by

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu_c e^*(n)y(n)\mathbf{x}(n), \tag{11}$$

where $e(n) = e^{i2\pi\alpha n} - y^2(n)$ and μ_c is the step size. The minima of the nonquadratic cost function were analyzed in [6]. In a single-user AWGN channel, the minima correspond to the points where output SNR is maximized if the desired signal has nonzero conjugate cyclic autocorrelation for the given α and τ. In a noiseless multiuser channel, a single signal which satisfies the following conditions can be extracted:

$$\kappa_i(0) < 2\frac{|\rho_i^\alpha(0)|}{|\rho_k^\alpha(0)|}, \quad \forall k \neq i, \tag{12}$$

where $\kappa_k(0)$ is the kurtosis and $\rho_k^\alpha(\tau)$ is the normalized conjugate cyclic autocorrelation (NCCA) function defined by

$$\kappa_k(\tau) \triangleq \frac{m_{4k}(\tau)}{\{m_{2k}(0)\}^2}, \;\; \rho_k^\alpha(\tau) \triangleq \frac{m_{2k}^\alpha(\tau)}{m_{2k}(0)}, \tag{13}$$

where $m_{2k}(\tau) \triangleq \left\langle s_k(t)s_k^*(t-\tau)\right\rangle$ and $m_{4k}(\tau) \triangleq \left\langle |s_k(t)|^2|s_k(t-\tau)|^2\right\rangle$. If there are several signals which satisfy the conditions, the desired signal cannot always be extracted: this depends on the initial weight vector.

3. Modified CF Algorithm

3.1 Cost Function

The conditions in (12) imply that the CF algorithm cannot distinguish the desired signal from other signals whose kurtosis and NCCA for a given frequency are almost the same as those of the desired signal. Importantly, NCCA $\rho_k^\alpha(\tau)$ depends on delay τ as well as frequency α. Thus, the NCCA of the desired signal can differ from that of the other signals for some delay τ. This observation led us to exploit the delay as well as the frequency.

A modified cost function is defined by

$$J_m \triangleq \left\langle |e^{i2\pi\alpha t} - y(t)y(t-\tau)|^2\right\rangle. \tag{14}$$

Clearly, in the case of zero delay ($\tau = 0$), the new cost function reduces to the CF cost function in (10). This new cost function is based on the fact that cyclostationary signals $x(t)$ generate spectral lines when they pass through a quadratic transformation involving delays, i.e., $x(t)x(t-\tau)$ [3]. In practice, a nonzero delay is required in some cases. For example, when the pulse waveform $a_k(t)$ is a rectangular pulse, the zero delay product does not contain a spectral line at the harmonics of the symbol rate. On the other hand, the delay product generates spectral lines at the harmonics of the symbol rate for any of a number of nonzero delays τ.

The method of steepest descent which minimizes the cost function takes the form

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \mu_m \frac{\partial J_m}{\partial \mathbf{w}^*}, \tag{15}$$

where the complex derivative operator is defined by [10]

$$\frac{\partial}{\partial \mathbf{w}^*} \triangleq \left[\frac{\partial}{\partial w_{1R}} + i\frac{\partial}{\partial w_{1I}}, \cdots, \frac{\partial}{\partial w_{NR}} + i\frac{\partial}{\partial w_{NI}}\right]^T, \tag{16}$$

where $(\cdot)^T$ represents the transpose of a matrix and $(\cdot)_R$ and $(\cdot)_I$ represent the real and imaginary parts, respectively. In practice, the stochastic gradient algorithm, obtained by replacing the time-average with its instantaneous value, is used:

$$\begin{aligned}\mathbf{w}(n+1) \;=\; & \mathbf{w}(n) + \mu_m e^*(n)\{\mathbf{x}(n)y(n-\tau) \\ & + \mathbf{x}(n-\tau)y(n)\},\end{aligned} \tag{17}$$

where $e(n) = e^{i2\pi\alpha n} - y(n)y(n-\tau)$ and μ_m is the step size. The algorithm is very simple since the computational complexity in each iteration is proportional to the number of antennae N. In the following, we refer to this algorithm as the modified CF (MCF) algorithm.

3.2 Analysis of Stationary Points

We need to analyze the stationary points of our cost function since the function is a non-quadratic form of the array weights. To search for optimum solutions using the stochastic gradient algorithm, only the points corresponding to the optimum solutions should be minima. The analysis of stationary points presented here was done in a way similar to that in [6].

In the rest of this paper, we will use two key assumptions:

AS1) Signals $s_k(t)$ are zero-mean, symmetrical, and cyclostationary: i.e., $m_{2k}^\alpha(\tau) \neq 0$ for some α and τ.
AS2) Matrix $\mathbf{A}$ has full column rank so that all signals are separable.

AS1 is common in most forms of multiuser communication. AS2 can be satisfied if the number of signals K is less than the number of antennae N and the directions of arrival θ_k differ from each other.

Now, we study a noiseless multiuser channel. Given α and τ, the desired signal is assumed to have nonzero NCCA $|\rho_{21}^\alpha(\tau)| \neq 0$ and all signals can be classified into two sets:

$$\mathcal{A} = \{j : |\rho_{2j}^\alpha(\tau)| \neq 0\}, \;\; \bar{\mathcal{A}} = \{j : |\rho_{2j}^\alpha(\tau)| = 0\}. \tag{18}$$

The beamformer output can be written as

$$y(t) = \sum_{k=1}^{K} g_k s_k(t), \tag{19}$$

where $g_k \triangleq \mathbf{w}^H\mathbf{a}(\theta_k)$ is the magnitude of the beamformer response to the kth signal. In a desirable beamformer response, all signals except the desired signal are completely canceled; i.e., $|g_1| \neq 0$ and $g_k = 0$ for $k = 2, \cdots, K$.

For the sake of simplicity, we define new variables:

$$u_k \triangleq g_k\sqrt{m_{2k}(0)}, \;\; k = 1, 2, \cdots, K. \tag{20}$$

Taking into account assumption AS1 and in a way similar to [6], we can express J_m as a function of u_k:

$$\begin{aligned}J_m \;=\; & \left(\mathbf{u}^H\mathbf{u}\right)^2 + \left(\mathbf{u}^H\mathbf{\Phi}(\tau)\mathbf{u}\right)\left(\mathbf{u}^H\mathbf{\Phi}^*(\tau)\mathbf{u}\right) \\ & + \sum_{k=1}^{K}|u_k|^4\left(\kappa_k(\tau) - 1 - |\eta_k(\tau)|^2\right) \\ & - \mathbf{u}^T\mathbf{\Gamma}(\tau)\mathbf{u} - \mathbf{u}^H\mathbf{\Gamma}^*(\tau)\mathbf{u}^* + 1,\end{aligned} \tag{21}$$

where $\mathbf{u} \triangleq [u_1, \cdots, u_K]^T$, $\mathbf{\Phi}(\tau) \triangleq \mathrm{diag}[\eta_1(\tau), \cdots, \eta_K(\tau)]$, $\mathbf{\Gamma}(\tau) \triangleq \mathrm{diag}[\rho_1^\alpha(\tau), \cdots, \rho_K^\alpha(\tau)]$, and the normalized autocorrelation function $\eta_k(\tau) \triangleq m_{2k}(\tau)/m_{2k}(0)$.

The gradient of J_m with respect to $\mathbf{w}$ is

$$\frac{\partial J_m}{\partial \mathbf{w}^*} = \mathbf{AM}\frac{\partial J_m}{\partial \mathbf{u}}, \tag{22}$$

where $\mathbf{M} \triangleq \mathrm{diag}\left[\sqrt{m_{21}(0)}, \cdots, \sqrt{m_{2K}(0)}\right]$. It is shown in [6] that if assumption AS2 holds (i.e., the columns in $\mathbf{A}$ are linearly independent), the stationary points of J_m correspond to the point where the following gradient of J_m with respect to $\mathbf{u}$ vanishes:

$$\begin{aligned}\frac{\partial J_m}{\partial \mathbf{u}^*} &= 2\left(\mathbf{u}^H\mathbf{u}\right)\mathbf{u} + \left(\mathbf{u}^H\boldsymbol{\Phi}^*(\tau)\mathbf{u}\right)\boldsymbol{\Phi}(\tau)\mathbf{u} \\ &+ \left(\mathbf{u}^H\boldsymbol{\Phi}(\tau)\mathbf{u}\right)\boldsymbol{\Phi}^*(\tau)\mathbf{u} - 2\boldsymbol{\Gamma}^*(\tau)\mathbf{u}^* \\ &+2\begin{bmatrix} |u_1|^2u_1\alpha_1(\tau) \\ \vdots \\ |u_K|^2u_K\alpha_K(\tau)\end{bmatrix} = \mathbf{0}, \end{aligned} \tag{23}$$

where $\alpha_i(\tau) \triangleq \kappa_i(\tau) - 1 - |\eta_i(\tau)|^2$. The stationary points can be classified into four cases. In the following, we summarize the properties of the stationary points. The details are presented in the Appendix.

Case 1: All signals are canceled; i.e.,

$$u_k = 0 \;\; \forall k. \tag{24}$$

This is not a desirable solution. This point is always a saddle point.

Case 2: A single signal $s_i(t)$ with nonzero $|\rho_i^\alpha(\tau)|$ is extracted and the other signals are canceled; i.e.,

$$|u_i| \neq 0, \;\; i \in \mathcal{A}, \;\; u_k = 0, \; \forall k \neq i. \tag{25}$$

From (23), we have

$$|u_i|^2 = \frac{|\rho_i^\alpha(\tau)|}{\kappa_i(\tau)}, \quad \arg(u_i) = -\frac{1}{2}\arg(m_{2i}^\alpha(\tau)). \tag{26}$$

This is a desirable solution. A set of sufficient conditions for the points to be minima is

$$\kappa_i(\tau) < \frac{|\rho_i^\alpha(\tau)|}{|\rho_k^\alpha(\tau)|}e_{ki}(\tau), \quad \forall k \in \mathcal{A}, \; k \neq i, \tag{27}$$

where

$$e_{ij}(\tau) \triangleq 1 + \mathrm{Re}\{\eta_i^*(\tau)\eta_j(\tau)\}. \tag{28}$$

Case 3: A linear combination of signals from a set $\mathcal{M}$ which consists of signals with nonzero $|\rho_i^\alpha(\tau)|$ is extracted and the other signals are canceled; i.e.,

$$u_i \neq 0, \; i \in \mathcal{M}, \; u_k = 0, \; k \notin \mathcal{M}, \; \mathcal{M} \subseteq \mathcal{A}. \tag{29}$$

From (23), we have

$$\begin{aligned}|u_i|^2 &= \frac{|\rho_i^\alpha(\tau)| - \mathbf{u}^H\mathbf{u} - \mathrm{Re}\{(\mathbf{u}^H\boldsymbol{\Phi}^*(\tau)\mathbf{u})\eta_i(\tau)\}}{\alpha_i(\tau)}, \\ \arg(u_i) &= -\frac{1}{2}\arg(m_{21}^\alpha(\tau)), \; \forall i \in \mathcal{M}. \end{aligned} \tag{30}$$

This is not a desirable solution. We can group all signals which belong to $\mathcal{M}$ into two sets $\mathcal{I}^+$ and $\mathcal{I}^-$ such that

$$\mathcal{I}^+ = \{j : f_j(\tau) > 0\}, \;\; \mathcal{I}^- = \{j : f_j(\tau) < 0\}, \tag{31}$$

where

$$f_j(\tau) \triangleq \left\{1 + \sum_{i\in\mathcal{M}} d_i(\tau)e_{ij}(\tau)\right\}\left\{1 + d_j(\tau)e_{jj}(\tau)\right\}, \tag{32}$$

where $d_i(\tau) \triangleq 1/\alpha_i(\tau)$. Then, a sufficient condition for the points to be saddle points is that there exists a signal $j \in \mathcal{I}^+$ such that

$$\sum_{i\in\mathcal{M}} d_i(\tau)e_{ij}(\tau)\left(\frac{|\rho_i^\alpha(\tau)|}{|\rho_j^\alpha(\tau)|} - 1\right) > 1, \tag{33}$$

or there exists a signal $j \in \mathcal{I}^-$ such that

$$\sum_{i\in\mathcal{M}} d_i(\tau)e_{ij}(\tau)\left(\frac{|\rho_i^\alpha(\tau)|}{|\rho_j^\alpha(\tau)|} - 1\right) < 1. \tag{34}$$

Case 4: A linear combination of signals from a set $\mathcal{M}$ which includes at least one signal with zero $|\rho_i^\alpha(\tau)|$ is extracted and the other signals are canceled; i.e.,

$$u_i \neq 0, \; i \in \mathcal{M}, \; u_k = 0, \; k \notin \mathcal{M}, \; \mathcal{M} \cap \bar{\mathcal{A}} \neq \emptyset. \tag{35}$$

This is not a desirable solution. This point is always a saddle point.

It is worth noting that the conditions derived above depend on delay τ. For example, the conditions in (27) for the points corresponding to single signal extraction to be minima depend on τ, while the corresponding conditions in (12) for the CF algorithm do not. The above analysis can be regarded as a generalization of that presented in [6] for cases of non-zero τ. When $\tau = 0$, the above results are basically the same as those in [6]. A notable difference can be found in a sufficient condition derived in Case 3 where the sign of $f_j(\tau)$ is taken into account.

If there is only one signal whose NCCA is nonzero, Case 3 never occurs and the Hessian in Case 2 is always positive definite as explained in the Appendix. Consequently, the signal is always extracted.

In the case where there are several signals whose NCCA is nonzero, one of the signals satisfying the conditions (27) can be extracted. The dependence of the conditions on the delay τ implies that the delay is useful to prevent the capture of undesired signals. In the next section, we will show how the MCF algorithm deals with the capture problem when applied to CDMA systems.

The results shown in this section only demonstrate that the MCF algorithm can be globally convergent in noiseless multiuser channels. Nevertheless, as shown by the simulation results in Sect. 6, the MCF algorithm also works well in noisy multiuser channels.

As for a single-user AWGN channel, we have analyzed the properties of the minima. This analysis proceeded in a way similar to that in [6], where the case of $\tau = 0$ was analyzed. Although the details are not presented here, we could prove that the MCF algorithm with $\tau \neq 0$ can provide the maximum SNR solution regardless of the initial array weights.

The conditions derived above depend only on statistics $\rho_k^\alpha(\tau), \eta_k(\tau)$, and $\kappa_k(\tau)$ which are normalized by the signal power $m_{2k}(0)$ or $\{m_{2k}(0)\}^2$. This implies that the property of stationary points is independent of the amplitudes A_k.

4. Application to CDMA Systems

4.1 CDMA Systems

In DS/CDMA systems, several users transmit at the same time and frequency band, but use distinct preassigned spreading codes. When correlators are used for demodulation, nonzero cross-correlation among the spreading codes leads to multiple-access interference (MAI) which degrades system performance. In practice, the spreading codes are chosen so that the cross-correlations are very small, such as with Gold codes. However, even if the cross-correlation is small, when the amplitude of the desired signal is much smaller than that of other signals (i.e., the near-far problem), MAI leads to significant performance degradation. Adaptive beamforming is a promising technique to compensate for MAI, since it can spatially separate the desired user's signal from other signals.

Let us consider BPSK/DS/CDMA systems. The transmitted signal of the kth user is given in (7). The data $\{I_{kn}\}$ are independent and equally likely to be +1 or −1. For different users, the sequences are mutually independent. The signature waveform of the kth user is given by

$$a_k(t) = \sum_{l=0}^{L_k-1} a_{kl} P_{T_{ck}}(t - lT_{ck}), \tag{36}$$

where $a_{kl} \in \{+1, -1\}, l = 0, 1, \cdots, L_k - 1$ is a spreading code of the kth user, L_k is the length of the spreading code, $P_{T_{ck}}(t)$ is a rectangular chip waveform, $P_{T_{ck}}(t) = 1$ for $t \in [0, T_{ck}]$ and 0 for $t \notin [0, T_{ck}]$, and $T_{ck} = T_k/L_k$ is the chip duration. The signals $\{s_k(t)\}$ satisfy assumption AS1, and thus are cyclostationary with frequency $\alpha = 2f_{ck} \pm l/T_k, l = 0, 1, \cdots$.

4.2 Sufficient Conditions

In this section, we show that the MCF algorithm always extracts a single signal in the noiseless BPSK/DS/CDMA system under certain conditions.

We define the partial autocorrelation function as

$$\phi_k(\tau) \triangleq \frac{1}{T_k} \int_\tau^{T_k} a_k(t) a_k(t - \tau) dt. \tag{37}$$

For convenience and without loss of generality, let signals in $\mathcal{A}$ be renumbered in the order of decreasing $|\rho_i^\alpha(\tau)|$ such that $|\rho_1^\alpha(\tau)| \geq |\rho_2^\alpha(\tau)| \geq \cdots \geq |\rho_{|\mathcal{A}|}^\alpha(\tau)|$, and define the corresponding set $\tilde{\mathcal{A}} = \{1, 2, \cdots, |\mathcal{A}|\}$. The following lemma provides a set of sufficient conditions for undesired stationary points, corresponding to Case 3 in the previous section, to be saddle points.

Lemma 1: The stationary points where a linear combination of signals in $\tilde{\mathcal{A}}$ is extracted are saddle points if

$$\text{C1)} \quad \phi_i^2(\tau) < 1 + \phi_i(\tau)\phi_j(\tau), \quad \forall i, j \in \tilde{\mathcal{A}}, \tag{38}$$

$$\text{C2)} \quad \frac{|\rho_i^\alpha(\tau)|}{|\rho_{i+1}^\alpha(\tau)|} > \frac{1 + \phi_i(\tau)\phi_{i+1}(\tau)}{1 + \phi_i(\tau)\phi_{i+1}(\tau) - \phi_i^2(\tau)}, \quad i = 1, 2, \cdots, |\tilde{\mathcal{A}}| - 1. \tag{39}$$

Proof: Since $|s_k(t)|^2 = A_k^2$, we have

$$m_{2k}(0) = \left\langle |s_k(t)|^2 \right\rangle = A_k^2 \tag{40}$$

$$m_{4k}(\tau) = \left\langle |s_k(t)|^2 |s_k(t - \tau)|^2 \right\rangle = A_k^4. \tag{41}$$

Taking into account the independent hypothesis on $\{I_{km}\}$, we get $m_{2k}(\tau) = A_k^2 \phi_k(\tau) e^{i2\pi f_{ck}\tau}$, and therefore

$$\eta_k(\tau) = \phi_k(\tau) e^{i2\pi f_{ck}\tau}. \tag{42}$$

Since $\kappa_k(\tau) = 1$, we have $\alpha_k(\tau) = -\phi_k^2(\tau)$, and thus

$$d_k(\tau) = -1/\phi_k^2(\tau). \tag{43}$$

Let $\mathcal{M}$ be a subset of $\tilde{\mathcal{A}}$. From (42) and (43), we have

$$1 + \sum_{i \in \mathcal{M}} d_i(\tau) \left(1 + \mathrm{Re}\{\eta_i^*(\tau)\eta_j(\tau)\}\right) = 1 - \sum_{i \in \mathcal{M}} \frac{1 + \phi_i(\tau)\phi_j(\tau)}{\phi_i^2(\tau)}, \quad j \in \mathcal{M}. \tag{44}$$

If each term of the sum is greater than one, (44) is negative for a given set $\mathcal{M}$. Thus, a set of sufficient conditions for (44) to be negative for any set $\mathcal{M}$ is

$$\frac{1 + \phi_i(\tau)\phi_j(\tau)}{\phi_i^2(\tau)} > 1, \quad \forall i, j \in \tilde{\mathcal{A}}. \tag{45}$$

Then, since

$$1 + d_k(\tau)(1 + |\eta_k(\tau)|^2) = -\frac{1}{\phi_k^2(\tau)} < 0, \tag{46}$$

$f_j(\tau)$ is positive for any set $\mathcal{M}$ and all $j \in \mathcal{M}$. This implies $\mathcal{M} = \mathcal{I}^+$.

The next step is to derive sufficient conditions for (33). Given a set $\mathcal{M}$ and $j \in \mathcal{M}$, the inequality in (33) holds if

$$d_i(\tau) e_{ij}(\tau) \left(\frac{|\rho_i^\alpha(\tau)|}{|\rho_j^\alpha(\tau)|} - 1 \right) > 1, \quad \forall i \in \mathcal{M}. \tag{47}$$

Since $d_i(\tau) = -1/\phi_i^2(\tau) < 0$ and $e_{ij}(\tau) = 1 + \phi_i(\tau)\phi_j(\tau) > 0$, they can be rewritten as

$$\frac{|\rho_i^\alpha(\tau)|}{|\rho_j^\alpha(\tau)|} - 1 < \frac{-\phi_i^2(\tau)}{1 + \phi_i(\tau)\phi_j(\tau)}, \quad \forall i \in \mathcal{M}. \tag{48}$$

Clearly, a set of sufficient conditions for (48) to be satisfied for any set $\mathcal{M}$ is

$$\frac{|\rho_{i+1}^\alpha(\tau)|}{|\rho_i^\alpha(\tau)|} - 1 < \frac{-\phi_i^2(\tau)}{1 + \phi_i(\tau)\phi_{i+1}(\tau)} \tag{49}$$

for $i = 1, 2, \cdots, |\tilde{\mathcal{A}}| - 1$. □

A set of conditions C1 is easily satisfied when long length spreading codes are employed because their autocorrelation functions $\phi_i(\tau)$ take very small values compared to one. As for C2, roughly speaking, when the values of $\phi_i(\tau)$ are negligible, these conditions can be rewritten as

$$|\rho_i^\alpha(\tau)| > |\rho_{i+1}^\alpha(\tau)|, \quad i = 1, \cdots, |\tilde{\mathcal{A}}| - 1. \tag{50}$$

This requires that the NCCAs differ from each other. As shown in the next section, this can be accomplished by choosing α and τ carefully.

The following theorem provides a set of sufficient conditions for the desired points to be minima.

Theorem 2: Suppose that conditions C1 and C2 are satisfied. The minima of the MCF cost function correspond to the points where a single signal $s_i(t), i \in \mathcal{A}$, is extracted if

$$\text{C3)} \quad \kappa_i(\tau) < \frac{|\rho_i^\alpha(\tau)|}{|\rho_k^\alpha(\tau)|}\left(1 + \phi_i(\tau)\phi_k(\tau)\right), \quad \forall k \in \mathcal{A}, k \neq i. \tag{51}$$

Proof: From Lemma 1, C1 and C2 ensure that the stationary points where a linear combination of signals is extracted are saddle points. It is clear from the discussion in 3.2 that all other undesired stationary points are saddle points. Substituting $e_{ki}(\tau) = 1 + \phi_k(\tau)\phi_i(\tau)$ into (27), we get (51). Clearly, they are sufficient to ensure that the desired points are minima. □

Conditions C3 can be satisfied for only the desired signal $s_1(t)$, if we choose α and τ such that the NCCA of the desired signal $|\rho_1^\alpha(\tau)|$ is significantly larger than that of the other signals.

To extract only the desired signal, delay τ must be chosen so as to satisfy conditions C1 and C2 for all signals and C3 for only the desired signal. To evaluate these conditions, statistics of all signals should be known. If some of the statistics are unknown, the delay cannot always be chosen properly. When a wrong delay is used, either an undesired signal or a linear combination of signals is extracted. A strategy to deal with this situation would be to choose τ such that $|\rho_i^\alpha(\tau)|$ of the desired signal is sufficiently large.

In the above analysis, no synchronization between users is assumed. Thus, the MCF algorithm can be applied to both synchronous and asynchronous CDMA systems with no modification. Both synchronous and asynchronous CDMA systems are considered in our simulation.

The proposed technique does not need symbol timing synchronization between the transmitter and receiver, unlike conventional blind beamformers for CDMA systems [11]. The synchronization can be carried out after the beamforming. Since the signal after beamforming contains only a small MAI component, simple synchronization techniques such as DLL can be successfully used.

5. Simulation Results

Computer simulations were carried out to show that the proposed MCF algorithm can extract the desired signal by appropriately choosing the delay. The performance of the proposed algorithm was compared with that of the CF algorithm [6] and SCORE algorithm [4] implemented with a gradient algorithm. In our simulation examples, we considered a uniform five-element array with half-wavelength spacing. BPSK/DS/CDMA systems using signature waveforms in (36) were considered. Gold sequences were used as the spreading codes. The length of the desired user's code was 31. All signals had the same chip rate, $1/T_c$. The received signals were sampled at a rate five times faster than the chip rate. We set our initial array weights to zero except w_1. The received SNR was fixed at 20 dB. The normalized carrier frequency and amplitude of the desired signal were $f_{c1} = 0.1$ and $A_1 = 1$. The desired signal arrived at the array from angle $\theta_1 = 30°$. The performance measure was the output signal to the interference and noise ratio (SINR) obtained by averaging over 10 trials as

$$\text{SINR} = \frac{1}{10}\sum_{i=1}^{10} \frac{|A_1 \mathbf{w}_i^H \mathbf{a}(\theta_1)|^2}{\mathbf{w}_i^H \left(\sum_{j\neq 1} A_j^2 \mathbf{a}(\theta_j)\mathbf{a}^H(\theta_j) + \sigma^2 \mathbf{I}\right)\mathbf{w}_i},$$

where $\mathbf{w}_i$ is the weight vector of the ith trial and σ^2 is the noise power. Channels were assumed to be time-invariant. We did not consider multipath channels. The control vector of the SCORE algorithm was $\mathbf{c} = [1, 0, \cdots, 0]^T$. The step size μ_m of the MCF algorithm was large enough to converge, and that of the SCORE and CF algorithms was chosen such that their convergence rate became almost the same as that with the MCF algorithm. Frequency parameter α and delay parameter τ of the MCF algorithm were chosen so as to satisfy conditions C1 and C2 for all signals and C3 for only the desired signal, and the same parameters were used for the SCORE algorithm. Unless otherwise stated, the frequency parameter of the CF algorithm was the same as that of the MCF algorithm.

5.1 Examples of Normalized Conjugate Cyclic Autocorrelation Function

First, we examined the NCCA function of signals to determine α and τ satisfying C1, C2, and C3. Figure 1 shows the magnitude of the NCCA function for three signals employing the Gold code of length 31 when cycle frequencies were chosen as $\alpha = 2f_{c1}$ and $2f_{c1} + 1/T_1$. Note that when $\alpha = 2f_{c1}$, the NCCA function was identical to the partial autocorrelation function: $\rho_i^\alpha(\tau) = \phi_i(\tau)$. The results clearly show that when $\alpha = 2f_{c1}$, the autocorrelation functions take values smaller than one if $\tau \neq 0$. Actually, we could confirm that these signals satisfy condition C1 for all $\tau \neq 0$. We also observed that although three NCCAs were the same when $\tau = 0$, they differed from each other and one of them was significantly larger than the others at a certain delay when $\tau \neq 0$. Actually, we found several delays satisfying conditions C2 and C3.

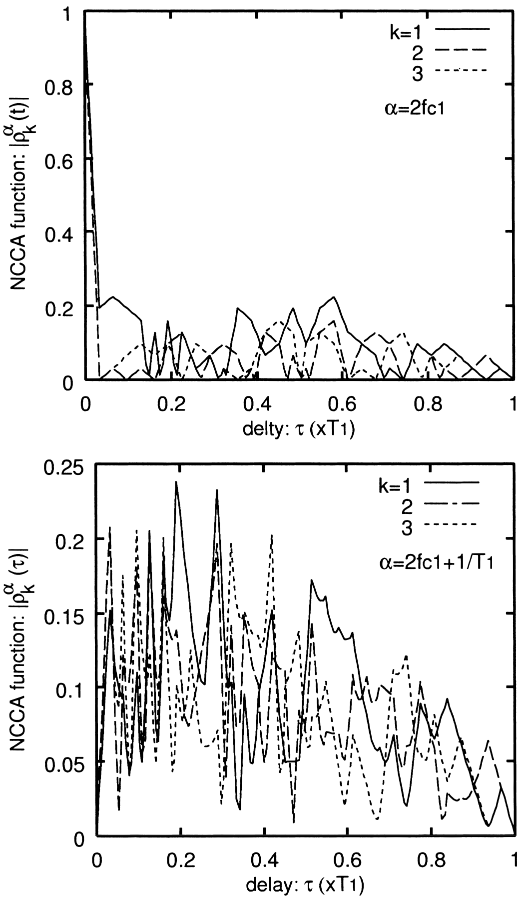

Fig. 1 Magnitude of normalized conjugate cyclic autocorrelation function for $\alpha = 2f_{c1}$ and $2f_{c1} + 1/T_1$.

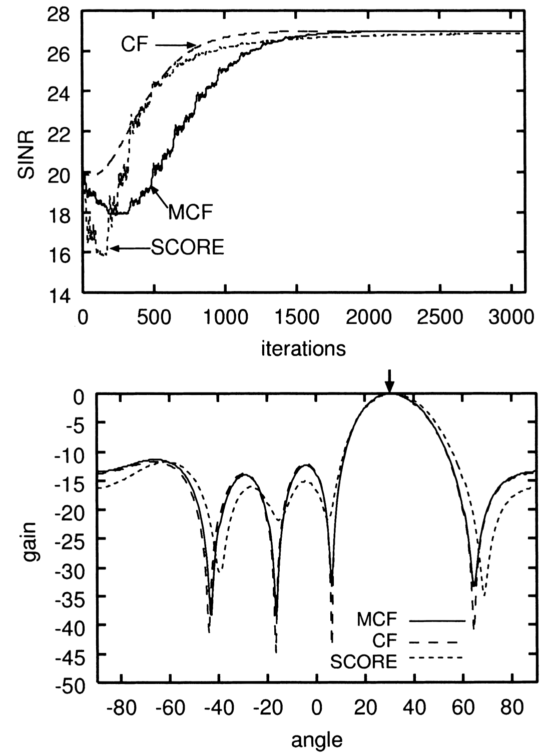

Fig. 2 Time evolution of the SINR and radiation pattern with the SCORE, CF, and MCF algorithms in a single-user channel.

5.2 Single-User AWGN Channel

In the first example, a single-user AWGN channel was considered to show that all three algorithms can provide the maximum SNR solution. The frequency and delay parameters were chosen as $\alpha = 2f_{c1}$ and $\tau = 0.071T_1$. The step sizes for the MCF, CF, and SCORE algorithms were chosen as $\mu_m = 10^{-3}, \mu_c = 10^{-3}$, and $\mu_s = 5 \times 10^{-5}$, respectively. The evolutions of the output SINR and the final radiation patterns for the three algorithms are shown in Fig. 2. The theoretically achievable SNR was 27 dB. As expected, all algorithms successfully achieved the maximum SNR.

5.3 Extraction of a Signal with a Distinct Cycle Frequency

In the second example, we considered a three-user synchronous CDMA system where a cycle frequency of the desired signal is distinct from that of the other signals. The carrier frequency of the desired signal was $f_{c1} = 0.1$, and that of two other signals was $f_{c2} = f_{c3} = 0.12$. The frequency and delay parameters were chosen as $\alpha = 2f_{c1}$ and $\tau = 0.013T_1$. The two interfering signals arrived at the antennae from angles $\theta_2 = -45°$ and $\theta_3 = 0°$. All signals had the same code length and signal amplitude. The step size of all algorithms was 10^{-4}. Figure 3 shows the time evolution of the SINR and the final radiation patterns. In this case, since the desired signal is the only signal which is cyclostationary with α, we expected the three algorithms to work well. As expected, all algorithms could extract the desired signal and suppress other signals.

5.4 Extraction of a Signal with a Common Cycle Frequency

In the third example, we considered a three-user dual-rate synchronous CDMA system to show that the MCF algorithm can extract the desired signal even in the presence of an interfering signal with the same cycle frequency as the desired signal. The symbol rate of the desired and second signal was $T_1 = T_2 = 31T_c$. The third signal used a Gold code of length 15, thus its symbol rate was $T_3 = 15T_c$. Two interfering signals arrived at the antennae from angles $\theta_2 = -45°$ and $\theta_3 = 60°$. All signals had the same carrier frequency and signal amplitude. The frequency and delay parameters of the MCF and SCORE algorithm were chosen as $\alpha = 2f_{c1} + 1/T_1$ and $\tau = 0.2T_1$. Since NCCA $|\rho_i^f(0)| = 0$ at $f = 2f_{c1} + 1/T_1$, the frequency parameter of the CF algorithm was set to $\alpha = 2f_{c1}$. The time evolution of the SINR and final antenna pattern are depicted in Fig. 4 where step sizes were $\mu_m = 2 \times 10^{-5}$, $\mu_c = 2 \times 10^{-5}$, and $\mu_s = 10^{-6}$. The CF algorithm captured the second signal which was cyclostationary at $2f_{c1}$ and suppressed the desired signal. The SCORE algorithm could suppress the third signal which had

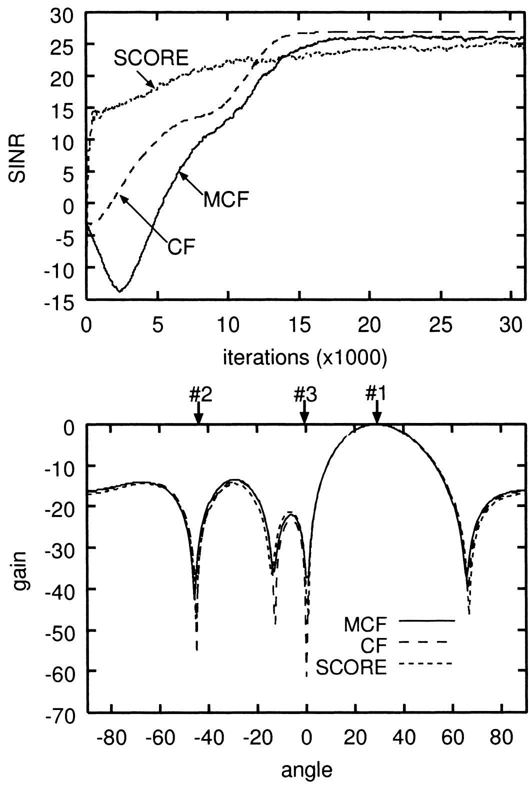

Fig. 3 Time evolution of the SINR and radiation pattern with the SCORE, CF, and MCF algorithms when the first signal has a distinct cycle frequency.

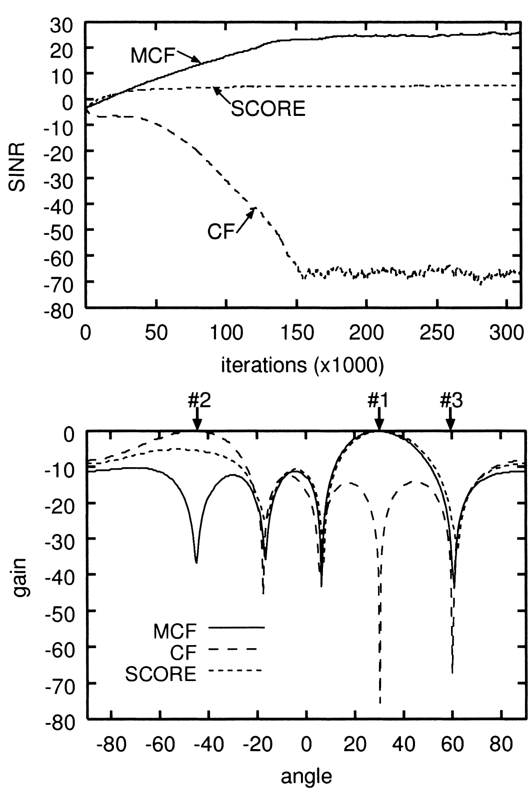

Fig. 4 Time evolution of the SINR and radiation pattern with the SCORE, CF, and MCF algorithms in the presence of an interfering signal with a common cycle frequency.

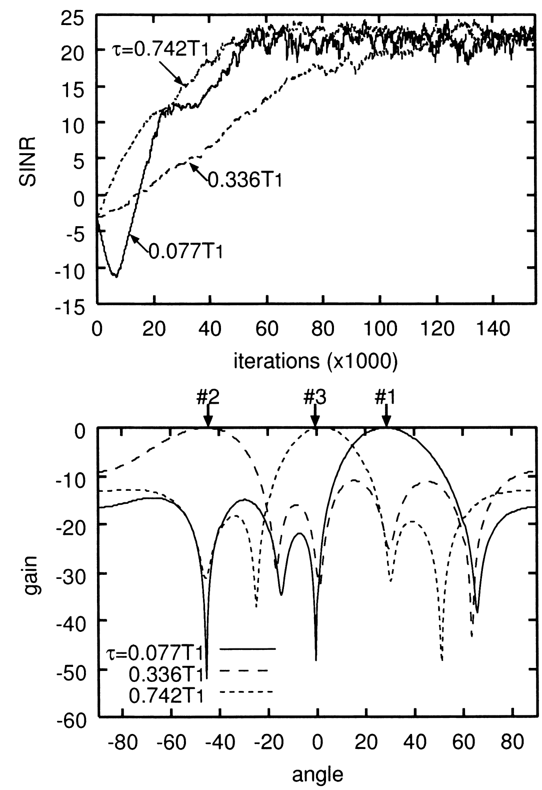

Fig. 5 Effect of delay choice on time evolution of the SINR and radiation pattern with the MCF algorithm when all signals have the same cycle frequency.

zero NCCA at $\alpha = 2f_{c1} + 1/T_1$, but could not suppress the second signal. The MCF algorithm extracted the desired signal and successfully suppressed the interfering signals.

5.5 Signal Selectivity Based on Delay

In the fourth example, we considered a three-user synchronous CDMA system where all signals have the same cycle frequency to show how the choice of the delay affects the behavior of the MCF algorithm. All signals had the same carrier frequency, code length, and amplitude. The frequency parameter and step size were chosen as $\alpha = 2f_{c1}$ and $\mu_m = 10^{-4}$, respectively. The angles of arrival were the same as in the second example. In Fig. 5, the time evolution of the SINR and the final radiation patterns in the cases of $\tau = 0.077T_1, 0.336T_1$, and $0.742T_1$ are shown, where the SINR was computed by treating $s_2(t)$ as the desired signal instead of $s_1(t)$ in the case of $\tau = 0.336T_1$, as $s_3(t)$ was in the case of $\tau = 0.742T_1$. As is clearly shown, the MCF beamformer can be used to extract any signal by choosing an appropriate delay.

5.6 Near-Far Situation

In the fifth example, we considered a three-user synchronous CDMA system with a near-far problem to show that the MCF algorithm works well even if strong interference exists. The amplitudes of the second and third

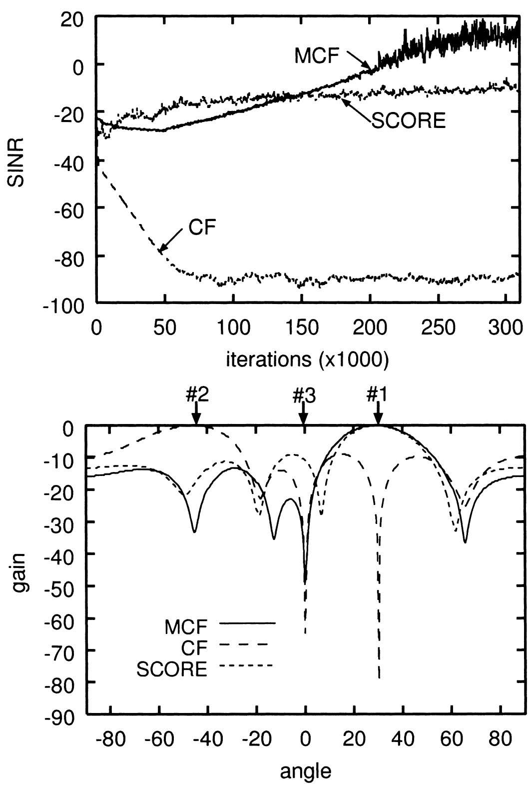

Fig. 6 Time evolution of the SINR and radiation pattern with the SCORE, CF, and MCF algorithms in the presence of two strong interfering signals.

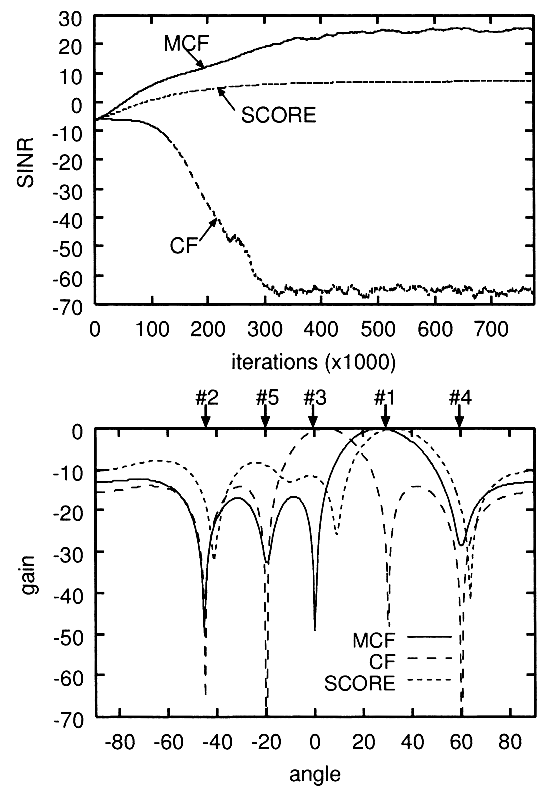

Fig. 7 Time evolution of the SINR and radiation pattern with the SCORE, CF, and MCF algorithms in a five-user asynchronous CDMA system.

signals were ten times as large as that of the desired signal. The chosen frequency parameter, delay parameter, and step sizes were, respectively, $\alpha = 2f_{c1}$, $\tau = 0.077T_1$, and $\mu_m = 10^{-5}, \mu_c = 10^{-5}, \mu_s = 10^{-6}$. The other parameters were the same as in the previous example. Figure 6 shows the evolution of the SINR and final antenna pattern. The CF algorithm captured the second signal and could not extract the desired signal, whereas the two other algorithms extracted the desired signal. However, the achievable SINR with the SCORE algorithm was lower than that with the MCF algorithm because the SCORE could not sufficiently suppress the strong interference.

5.7 Five-User Asynchronous CDMA System

In the sixth example, we considered a five-user asynchronous CDMA system to show that the MCF algorithm works well in the presence of many signals in an asynchronous system. The chosen cycle frequency, delay, and step sizes were, respectively, $\alpha = 2f_{c1}$, $\tau = 0.077T_1$, and $\mu_m = 10^{-5}, \mu_c = 10^{-5}, \mu_s = 2 \times 10^{-7}$. The undesired signals came from $\theta_2 = -45°, \theta_3 = 0°, \theta_4 = 60°$, and $\theta_5 = -20°$. All signals had the same carrier frequency, code length, and signal amplitude. In Fig. 7, the time evolution of the SINR and the final radiation pattern are shown. The CF algorithm captured the third signal, whereas the SCORE algorithm could not sufficiently suppress the undesired signals. It is clear that

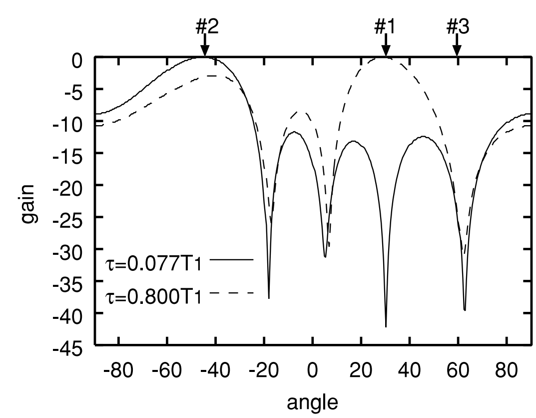

Fig. 8 Radiation pattern with the MCF algorithm with wrong choices for τ.

the MCF algorithm can extract the desired signal and suppress the interfering signals even in an asynchronous system.

5.8 Effect of Wrong Delay Choices

Finally, we show examples of a wrong choice for the delay. Figure 8 shows the radiation pattern where all parameters except the delay are the same as in Fig. 4. In both curves, the third signal, which has zero NCCA at α, was suppressed. However, an undesired signal was extracted when

$\tau = 0.077T_1$ and a combination of the first and second signal was extracted when $\tau = 0.800T_1$. These results verify the analytical results given in Sect. 4.

From the results presented in this section, we conclude that the CF algorithm, which does not use the delay, has a limited capability for extracting the desired signal, while the exploitation of the non-zero delay improves the capability.

Since the main purpose of our simulation was to show how the MCF algorithm avoids the capture problem, the step size, α and τ were not optimized to obtain the fastest possible convergence. According to our preliminary experiments, the MCF algorithm tends to show slower convergence than the other algorithms. This slow convergence may be due to the small magnitude of the NCCA upon which the MCF algorithm relies. In fast varying channels, slow convergence leads to serious performance degradation, so improved convergence speed is required. While recursive least-squares type algorithms [5] could provide faster convergence, this would lead to greater computational complexity. This issue will need to be further considered in the future.

6. Conclusion

We have described a simple adaptive beamforming technique based on higher-order statistics regarding cyclostationary signals. The proposed method exploits the delay as well as cycle frequency. This greatly reduces the possibility of capturing undesired signals. We derived a set of conditions which ensure that the MCF algorithm in noiseless multiuser systems extracts only a single signal. Moreover, we showed that these conditions can be satisfied in CDMA system applications and the desired signal successfully extracted.

The proposed method replaces the zero-delay product in the CF algorithm [6] with a delay product. Further generalization is possible by replacing it with a linear combination of delay products. This generalization may improve the convergence speed and reduce the capture problem. Further comprehensive analysis will be the subject of our ongoing work.

Though we did not consider multipath propagation situations since such an analysis would be very complicated, the performance of the proposed algorithm in multipath channels will be considered in the future.

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Appendix: Evaluation of the Hessian

To examine the stationary points of the cost function, first of all, we determine stationary points where the gradient of the cost function becomes zero. Then, we examine whether these points are minima by evaluating the positive definiteness of the Hessian at the point, which is defined by

$$\begin{aligned}\mathbf{H}_{\mathbf{W}} J_m &\triangleq \begin{bmatrix} \frac{\partial}{\partial \mathbf{W}}\left(\frac{\partial J_m}{\partial \mathbf{W}^*}\right)^T & \frac{\partial}{\partial \mathbf{W}}\left(\frac{\partial J_m}{\partial \mathbf{W}}\right)^T \\ \frac{\partial}{\partial \mathbf{W}^*}\left(\frac{\partial J_m}{\partial \mathbf{W}^*}\right)^T & \frac{\partial}{\partial \mathbf{W}^*}\left(\frac{\partial J_m}{\partial \mathbf{W}}\right)^T \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{E}_{\mathbf{W}} J_m & \mathbf{S}_{\mathbf{W}} J_m \\ \mathbf{S}^*_{\mathbf{W}} J_m & \mathbf{E}^*_{\mathbf{W}} J_m \end{bmatrix}. \end{aligned} \tag{A$\cdot$1}$$

Let $\mathbf{C} \triangleq \mathbf{AM}$. Then, the Hessian with respect to $\mathbf{w}$ can be written as

$$\mathbf{H}_{\mathbf{W}} J_m = \begin{bmatrix} \mathbf{C}^* & \mathbf{0} \\ \mathbf{0} & \mathbf{C} \end{bmatrix} \mathbf{H}^T_{\mathbf{u}} J_m \begin{bmatrix} \mathbf{C}^T & \mathbf{0} \\ \mathbf{0} & \mathbf{C}^H \end{bmatrix}. \tag{A$\cdot$2}$$

$\mathbf{H}_{\mathbf{u}} J_m$ has the same positive definiteness as $\mathbf{H}_{\mathbf{W}} J_m$ since they are congruent matrices [12]. The matrices $\mathbf{E}_{\mathbf{u}} J_m$ and $\mathbf{S}^*_{\mathbf{u}} J_m$ in the Hessian become

$$\begin{aligned}\mathbf{E}_{\mathbf{u}} J_m &= 2(\mathbf{u}^H\mathbf{u})\mathbf{I} + 2\mathbf{u}^*\mathbf{u}^T \\ &\quad + \left(\mathbf{u}^H \mathbf{\Phi}^*(\tau)\mathbf{u}\right)\mathbf{\Phi}(\tau) + \mathbf{\Phi}^*(\tau)\mathbf{u}^*\mathbf{u}^T\mathbf{\Phi}(\tau) \\ &\quad + \left(\mathbf{u}^H \mathbf{\Phi}(\tau)\mathbf{u}\right)\mathbf{\Phi}^*(\tau) + \mathbf{\Phi}(\tau)\mathbf{u}^*\mathbf{u}^T\mathbf{\Phi}^*(\tau) \\ &\quad + 4\mathrm{diag}\left[|u_1|^2\alpha_1(\tau), \cdots, |u_K|^2\alpha_K(\tau)\right], \end{aligned} \tag{A$\cdot$3}$$

$$\mathbf{S}^*_{\mathbf{u}} J_m = 2\mathbf{u}\mathbf{u}^T + \mathbf{\Phi}^*(\tau)\mathbf{u}\mathbf{u}^T\mathbf{\Phi}(\tau)$$

$$+ \mathbf{\Phi}(\tau)\mathbf{u}\mathbf{u}^T\mathbf{\Phi}^*(\tau) - 2\mathbf{\Gamma}^*(\tau) + 2\mathrm{diag}\left[u_1^2\alpha_1(\tau), \cdots, u_K^2\alpha_K(\tau)\right]. \quad \text{(A· 4)}$$

Now, we evaluate the Hessian for the four cases.

Case 1: All signals are canceled; i.e., $u_k = 0, \forall k$. Then, the Hessian becomes

$$\mathbf{H_u}J_m = \begin{bmatrix} \mathbf{0} & -2\mathbf{\Gamma}(\tau) \\ -2\mathbf{\Gamma}^*(\tau) & \mathbf{0} \end{bmatrix}. \quad \text{(A· 5)}$$

Clearly, this matrix is indefinite and therefore this point is a saddle point.

Case 2: A single signal $s_i(t)$ in $\mathcal{A}$ is extracted and the other signals are canceled; i.e., $|u_i| \neq 0, i \in \mathcal{A}, u_k = 0, \forall k \neq i$. The nonzero responses u_i are given in (26). Then, we obtain

$$\mathbf{E_u}J_m = |u_i|^2\mathrm{diag}\left[2e_{1i}, \cdots, 4\kappa_i(\tau), \cdots, 2e_{Ki}\right], \quad \text{(A· 6)}$$

$$\mathbf{S_u^*}J_m = -2\mathrm{diag}\left[\left(\rho_1^\alpha(\tau)\right)^*, \cdots, \left(\rho_i^\alpha(\tau)\right)^* - \kappa_i(\tau)u_i^2, \cdots, (\rho_K^\alpha(\tau))^*\right]. \quad \text{(A· 7)}$$

Using a permutation matrix $\mathbf{P}$, the Hessian $\mathbf{H_u}J_m$ can be transformed into another matrix $\mathbf{Q_u}J_m = \mathbf{P}^H\mathbf{H_u}J_m\mathbf{P}$ with a block-diagonal structure, whose ith second-order square submatrix consists of the ith diagonal entries of $\mathbf{E_u}J_m$ and $\mathbf{S_u}J_m$:

$$\mathbf{\Delta}_i = \begin{bmatrix} 2\kappa_i(\tau)|u_i|^2 & (u_i^*)^2\kappa_i(\tau) - \rho_i^\alpha(\tau) \\ u_i^2\kappa_i(\tau) - \left(\rho_i^\alpha(\tau)\right)^* & 2\kappa_i(\tau)|u_i|^2 \end{bmatrix} \quad \text{(A· 8)}$$

and the $k(\neq i)$-th submatrices are

$$\mathbf{\Delta}_k = \begin{bmatrix} |u_i|^2 e_{ki}(\tau) & -\rho_k^\alpha(\tau) \\ -\left(\rho_k^\alpha(\tau)\right)^* & |u_i|^2 e_{ki}(\tau) \end{bmatrix}, \ k \neq i. \quad \text{(A· 9)}$$

The matrix $\mathbf{Q_u}J_m$ has the same positive definiteness as $\mathbf{H_u}J_m$ since they are congruent matrices.

The determinant of a block-diagonal matrix is the product of the determinants of the submatrices [13]; e.g., $|\mathbf{Q_w}J_m| = \Pi_{i=1}^K|\mathbf{\Delta}_i|$. The next step is to evaluate the determinant sign of the above submatrices. Since $\kappa_i(\tau) > 0$, the $(1, 1)$ entry of $\mathbf{\Delta}_i$ is positive. The determinant of $\mathbf{\Delta}_i$ becomes

$$|\mathbf{\Delta}_i| = 4|\rho_i^\alpha(\tau)|^2, \quad \text{(A· 10)}$$

and this is positive because $s_i(t)$ is assumed to have nonzero NCCA. As for $\mathbf{\Delta}_k$, if $\tau = 0$, the $(1, 1)$ entry is positive since $\eta_i(0) = 1$, thus $e_{ij}(0) = 1 + \mathrm{Re}\{\eta_i^*(0)\eta_j(0)\} = 2$. Moreover, even if $\tau \neq 0$, this is positive because $|\eta_i(\tau)| < 1$ and thus $e_{ij}(\tau) > 0$. In the case of $k \in \bar{\mathcal{A}}$, since $\rho_k^\alpha(\tau) = 0$, $|\mathbf{\Delta}_k| = |u_i|^4 e_{ki}^2(\tau) > 0$. In the case of $k \in \mathcal{A}$, $|\mathbf{\Delta}_k|$ is positive if and only if

$$\kappa_i(\tau) < \frac{|\rho_i^\alpha(\tau)|}{|\rho_k^\alpha(\tau)|}e_{ki}(\tau). \quad \text{(A· 11)}$$

Consequently, if there is one signal in $\mathcal{A}$, this point is always a minimum. If there are several signals in $\mathcal{A}$, the conditions in (27) are sufficient for this point to be a minimum.

Case 3: A linear combination of signals from a set $\mathcal{M}$ which consists of signals in $\mathcal{A}$ is extracted and the rest of the signals are canceled; i.e., $u_i \neq 0$, $i \in \mathcal{M}$, $u_k = 0$, $k \notin \mathcal{M}$, $\mathcal{M} \subseteq \mathcal{A}$. The nonzero responses u_i are given in (30). For a signal $j \in \mathcal{M}$, using a permutation matrix $\mathbf{P}_j$, the Hessian $\mathbf{H_u}J_m$ can be transformed into another matrix $\mathbf{Q_u}J_m = \mathbf{P}_j\mathbf{H_u}J_m\mathbf{P}_j$ whose first second-order submatrix can be written as

$$\mathbf{\Delta}_1 = \begin{bmatrix} (\mathbf{\Delta}_1)_{11} & (\mathbf{\Delta}_1)_{12} \\ (\mathbf{\Delta}_1)_{21} & (\mathbf{\Delta}_1)_{22} \end{bmatrix} \quad \text{(A· 12)}$$

where

$$(\mathbf{\Delta}_1)_{11} = 2\|\mathbf{u}\|^2 + 2|u_j|^2\left(1 + |\eta_j(\tau)|^2 + 2\alpha_j(\tau)\right) + 2\mathrm{Re}\{(\mathbf{u}^H\mathbf{\Phi}^*(\tau)\mathbf{u})\eta_j(\tau)\} = (\mathbf{\Delta}_1)_{22},$$

$$(\mathbf{\Delta}_1)_{12} = 2(u_j^*)^2\left(1 + |\eta_j(\tau)|^2 + \alpha_j(\tau)\right) - 2\rho_j^\alpha(\tau) = ((\mathbf{\Delta}_1)_{21})^*.$$

The matrices $\mathbf{H_u}J_m$ and $\mathbf{Q_u}J_m$ have the same positive definiteness since they are congruent matrices. Thus, it is sufficient to show that the sign of $(\mathbf{\Delta}_1)_{11}$ differs from that of $|\mathbf{\Delta}_1|$ for $\mathbf{H_u}J_m$ to be indefinite.

Since $e_{ij}(\tau) > 0$ and $\kappa_j(\tau) > 0$, the $(1, 1)$ entry is always positive

$$(\mathbf{\Delta}_1)_{11} = 2\sum_{\substack{i\in\mathcal{M} \\ i\neq j}} |u_i|^2 e_{ij}(\tau) + 4|u_j|^2\kappa_j(\tau) > 0. \quad \text{(A· 13)}$$

The determinant of $\mathbf{\Delta}_1$ becomes

$$|\mathbf{\Delta}_1| = 16|\rho_j^\alpha(\tau)|\left[|\rho_j^\alpha(\tau) - \|\mathbf{u}\|^2 + |u_j|^2 e_{jj}(\tau) - \mathrm{Re}\{(\mathbf{u}^H\mathbf{\Phi}^*(\tau)\mathbf{u})\eta_j(\tau)\}\right]. \quad \text{(A· 14)}$$

Substituting the following relation into (A· 14),

$$\|\mathbf{u}\|^2 + \mathrm{Re}\{(\mathbf{u}^H\mathbf{\Phi}^*(\tau)\mathbf{u})\eta_j(\tau)\} = \frac{\sum_{i\in\mathcal{M}} d_i(\tau)|\rho_i^\alpha(\tau)|e_{ij}(\tau)}{1 + \sum_{i\in\mathcal{M}} d_i(\tau)e_{ij}(\tau)}, \quad \text{(A· 15)}$$

where $d_i(\tau) = 1/\alpha_i(\tau)$, we can show that the determinant of $\mathbf{\Delta}_1$ is negative if and only if

$$|\rho_j(\tau)| < \frac{\sum_{i\in\mathcal{M}} d_i(\tau)|\rho_i^\alpha(\tau)|e_{ij}(\tau)}{1 + \sum_{i\in\mathcal{M}} d_i(\tau)e_{ij}(\tau)} \cdot \left(1 + d_j(\tau)e_{jj}(\tau)\right) - d_j(\tau)|\rho_j^\alpha(\tau)|e_{jj}(\tau). \quad \text{(A· 16)}$$

When $\{1 + \sum_{i\in\mathcal{M}} d_i(\tau)e_{ij}(\tau)\}\{1 + d_j(\tau)e_{jj}(\tau)\} > 0$ (in other words, $j \in \mathcal{I}^+$), the condition results in

$$\sum_{i\in\mathcal{M}} d_i(\tau)e_{ij}(\tau)\left(\frac{|\rho_i^\alpha(\tau)|}{|\rho_j^\alpha(\tau)|} - 1\right) > 1, \quad \text{(A· 17)}$$

otherwise (i.e., if $j \in \mathcal{I}^-$),

$$\sum_{i\in\mathcal{M}} d_i(\tau)e_{ij}(\tau)\left(\frac{|\rho_i^\alpha(\tau)|}{|\rho_j^\alpha(\tau)|} - 1\right) < 1. \quad \text{(A· 18)}$$

Consequently, if there exists $j \in \mathcal{I}^+$ such that condition

(A· 17) is satisfied or there exists $j \in \mathcal{I}^-$ such that condition (A· 18) is satisfied, this point is a saddle point.

Case 4: A linear combination of signals from a set $\mathcal{M}$ which includes at least one signal from $\bar{\mathcal{A}}$ is extracted and the rest of the signals are canceled; i.e., $u_i \neq 0$, $i \in \mathcal{M}$, $u_k = 0$, $k \notin \mathcal{M}$, $\mathcal{M} \cap \bar{\mathcal{A}} \neq \emptyset$. As in the previous case, the first second-order submatrix of the transformed Hessian is given in (A· 12). From (A· 13), the (1,1) entry of the matrix is positive. If we choose $\mathbf{P}_j$ such that $j \in \bar{\mathcal{A}}$, $|\rho_j^\alpha(\tau)| = 0$, and thus the determinant of $\mathbf{\Delta}_1$ becomes zero. Consequently, the Hessian is indefinite, and thus this point is always a saddle point.

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