LETTER

# Differential Constant Modulus Algorithm for Anchored Blind Equalization of AR Channels

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**SUMMARY** A blind equalizer which uses the differential constant modulus algorithm (DCMA) is introduced. An anchored FIR equalizer applied to a first-order autoregressive channel and updated according to the DCMA is shown to converge to the inverse of that channel regardless of the initial tap-weights and the gain along the direct path.

***key words:*** *blind signal processing, adaptive equalizer, CMA, stability analysis*

## 1. Introduction

In the field of digital communications, considerable interest has been directed towards blind adaptive equalizers that are capable of eliminating the effects of intersymbol interference (ISI) without requiring training sequences [1]. Although fractionally spaced blind equalizers are attractive because of their global convergent property, blind equalizers with inputs sampled at baud-rate remain useful and practical for communication channels that lack excess bandwidth [2]. In this letter, we focus on the blind equalization of autoregressive (AR) channels by FIR filters with inputs sampled at baud-rate because it can achieve the perfect equalization [3]–[5]. Anchored FIR equalizers are attractive due to their simplicity [3], [4]. Verdú et al. [3] have proposed the use of the output energy as a cost function for anchored equalizers. The anchored equalizer is able to exhibit global convergence when applied to AR channels. However, applying a stochastic gradient method to achieve fast tracking produces a strong fluctuation of tap-weights around the desired point. This results in degradation of the system's performance in terms of bit error rate. Kamel et al. [4] have proposed an anchored equalizer in which the CMA is applied. The fluctuations of this equalizer's tap-weights are relatively small. However, the equalizer is only able to exhibit global convergence in application to AR channels in which the gain along the direct path is greater than a certain critical value. In this letter, we consider the application of the DCMA [6] to the blind equalization problem as a way of overcoming the above-mentioned disadvantages of conventional equalizers.

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## 2. Problem Statement

Assume that the information data sequence $\{x_k\}$ consists of i.i.d. random variables chosen from the QPSK constellation with $|x_k| = 1$. We assume, moreover, that there is no phase offset. The received signal through an $N$th-order AR channel is sampled every $T_b$ which is the symbol period:

$$r_k = Gx_k + \sum_{i=1}^{N} a_i r_{k-i} \tag{1}$$

where $G$ is an arbitrary gain along the direct path and $a_i$ are the AR parameters. Note that the magnitude of $a_i$ is assumed to be less than unity. In our analysis, we assume that the level of noise on the channel is negligible. To recover $x_k$ from the received signal $r_k$ that has been corrupted by ISI, the received signal is passed through an FIR equalizer with output written as

$$y_k = \sum_{i=0}^{M} w_i^* r_{k-i} \tag{2}$$

where $w_i$ are the tap-weights, $M+1$ is the number of tap-weights and asterisk denotes complex conjugate. In an anchored equalizer [3], [4], in order to prevent the situation where the output of the equalizer is always zero, the first tap-weight of the equalizer is fixed to unity, $w_0 = 1$. Although the non-anchored equalizers using the well known CMA are not globally convergent for arbitrary initialization [7], the anchored equalizers can be globally convergent. It is clear that, if $M = N$ and $w_i^* = -a_i, i = 1, \cdots, M$, the anchored equalizer can recover the transmitted symbol, i.e. $y_k = Gx_k$. In the following description, $M$ is assumed to be equal to $N$.

## 3. DCMA Blind Equalizer

The DCMA operates in such a way as to make the magnitudes of successive equalizer outputs equal. The DCMA cost function is given by

$$J(\mathbf{w}) = \frac{1}{4} E\left[ ||y_k|^2 - |y_{k-1}|^2|^2 \right] \tag{3}$$

where $\mathbf{w} = (w_1, \cdots, w_N)^T$. If the magnitude of the equalizer outputs becomes constant such that $|y_k|^2 = |y_{k-1}|^2$, the cost function is minimized. Note that when ISI is canceled, $|y_k|^2 = |y_{k-1}|^2$.

We examine the shape of the cost function. Consider a first-order AR channel with $a_1 \neq 0$. For simplicity of analysis, we introduce a variable $u^* = w_1^* + a_1$ instead of $w_1$. The stationary points of $J(u)$ corresponds to those of $J(w_1)$ because the stationary points are the points where the gradient vanishes. Thus, we consider the stability of the stationary points of $J(u)$. Taking into account the i.i.d. hypothesis on $x_k$ and the relationship $|x_k|^2 = 1$, the gradient of the DCMA cost function is given by

$$\begin{aligned}\frac{\partial J(u)}{\partial u^*} &= \frac{1}{4}\frac{|G|^4}{1-|a_1|^4}\left\{8|a_1|^2|u|^2 - (1+|a_1|^2)\right.\\ &\quad \left.\cdot(2u^*a_1^* + ua_1) + 4(1+|a_1|^2)\right\}u. \qquad (4)\end{aligned}$$

It is possible to show that the inside of the braces is nonzero if $|a_1| < 1$. Thus, the gradient is zeroed if and only if $u = 0$, i.e., $w_1^* = -a_1$, by which ISI is completely canceled. The Hessian matrix at $u = 0$ is given by

$$\begin{aligned}\mathbf{Q}(0) &= \left.\begin{bmatrix} \frac{\partial^2 J(u)}{\partial u \partial u^*} & \frac{\partial^2 J(u)}{\partial u \partial u} \\ \frac{\partial^2 J(u)}{\partial u^* \partial u^*} & \frac{\partial^2 J(u)}{\partial u^* \partial u} \end{bmatrix}\right|_{u=0} \\ &= \frac{|G|^4(1+|a_1|^2)}{1-|a_1|^2}\mathbf{I}. \qquad (5)\end{aligned}$$

The Hessian matrix can also be shown to be positive definite if $|a_1| < 1$. Consequently, the cost function is convex. There are no undesired minima except the desired solution. ISI can thus be canceled if the cost function is minimized.

From (3), the stochastic gradient algorithm takes the form

$$\begin{aligned}w_i^{(k+1)} &= w_i^{(k)} - \mu(|y_k|^2 - |y_{k-1}|^2)\\ &\quad \cdot(y_k^* r_{k-i} - y_{k-1}^* r_{k-1-i}) \qquad (6)\end{aligned}$$

where $i = 1, \cdots, N$ and $\mu$ is the step-size parameter. When the weights approach the desired solution, it is reasonable to expect that the difference $|y_k|^2 - |y_{k-1}|^2$ will be very small since both $|y_k|^2$ and $|y_{k-1}|^2$ approach $|G|^2$. As a result, we would expect very little fluctuation in the values of weights.

## 4. Simulation Results

The DCMA equalizer was compared with two conventional anchored equalizers using the minimum energy algorithm (MEA) [3] and CMA [4]. The data was chosen from the QPSK constellation. In this simulation, initial weights were set be zero except for $w_0^{(0)} = 1$. For the DCMA, the step-size was selected to provide the best performance in terms of bit error rate after 100 iterations when $E_b/N_0$=10 dB, $E_b$ is the signal energy per bit and $N_0$ is the PSD of the channel noise which corrupts $r_k$. The step-sizes for the other algorithms were chosen so that all of the algorithms would converge at almost the same rate.

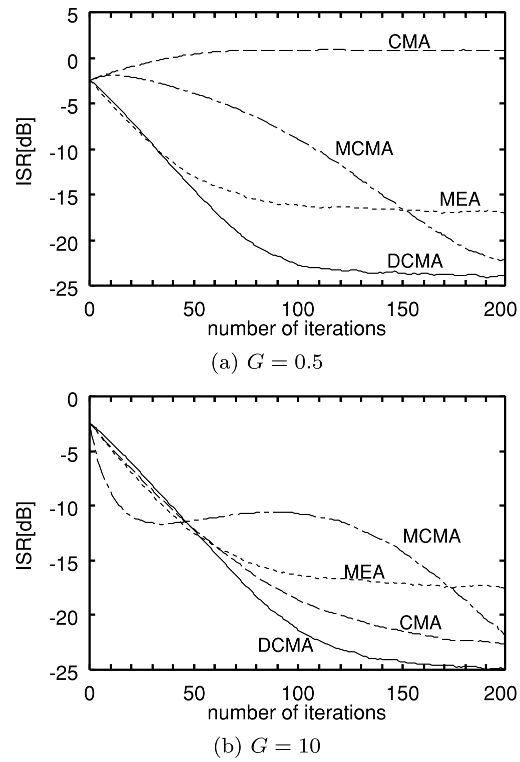
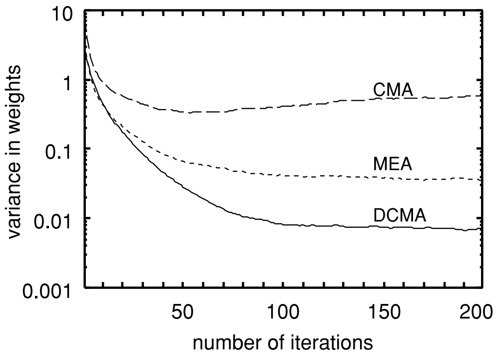



**Fig. 1** ISR against number of iterations.

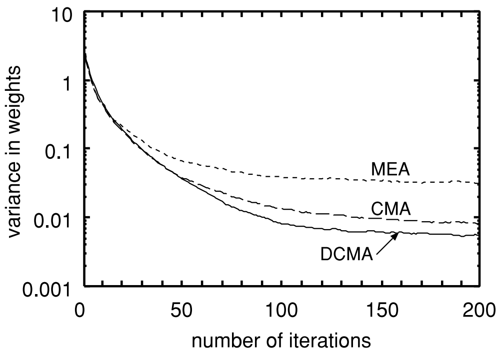
In the first experiment, the channel was a first-order AR system with $a_1 = 0.6$. Figure 1 is a plot of the time evolution of the interference-to-signal ratio (ISR) when $E_b/N_0$ =10 dB, and $G = 0.5$ and 10. ISR is defined as

$$\mathrm{ISR} = \frac{\sum_{i=0}^{10}|g_i|^2 - \max_i |g_i|^2}{\max_i |g_i|^2} \qquad (7)$$

where $g_i = \sum_{l=0}^{M} w_l h_{i-l}$ and $\{h_l\}$ is the impulse response of the channel. That the CMA fails to suppress ISI when $G = 0.5$, as has previously been reported [4], is clearly visible. On the other hand, the DCMA is shown to work well. This observation supports the analytical result presented in the previous section. Moreover, we are able to see that the ISR of the DCMA is lower than that of the MEA. We infer that this is due to the fluctuation of weights around the desired point. Figure 2 shows the variance in the weights. The variance in the weights is clearly lower for the DCMA than for the other algorithms. Although it is possible to reduce the variance in the weights of the MEA by using a smaller step-size, using small step-sizes is not possible when fast tracking is desired. Figure 3 shows bit error rate performance for $G = 0.5$ and 10. The DCMA outperforms the other algorithms even in the noisy channels because of the smaller fluctuation in its

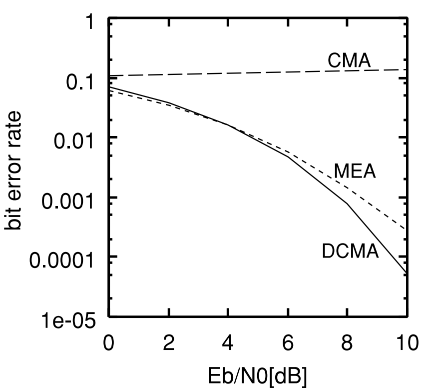


(a) $G = 0.5$

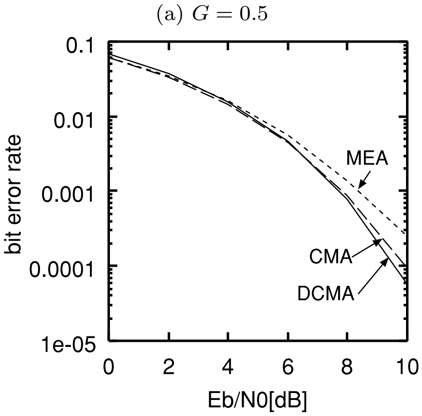


(b) $G = 10$

**Fig. 2** Variance in weights against number of iterations.



(a) $G = 0.5$



(b) $G = 10$

**Fig. 3** Bit error rate performance.

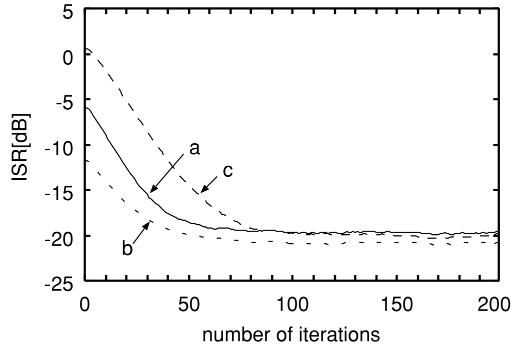



**Fig. 4** ISR by DCMA for second-order AR channels: (a) $a_1 = 0, a_2 = 0.25$, (b) $a_1 = 0.2, a_2 = 0.35$, (c) $a_1 = -0.4, a_2 = 0.4$.

weights.

So far, there have been many non-anchored CMA equalizers. As an example, a modified CMA (MCMA) recently proposed in [8] is considered. In Fig. 1, the ISR by the MCMA are shown. As for the parameters of the MCMA, we chose the step-size and weighting factor which provide the smallest ISR at 200 iterations. It can be observed that the convergence rate of the MCMA is relatively slow. Although the MCMA works well in this example, it should be noted that it is not clear whether the MCMA always converges regardless of the initial tap-weights or not.

In the second experiment, we considered three types of second-order AR channels. The parameters of the channels were (i) $a_1 = 0.2, a_2 = 0.35$, (ii) $a_1 = 0.0, a_2 = 0.25$ and (iii) $a_1 = 0.4, a_2 = -0.4$. The ISR performances of the DCMA when $G$=10 are shown in Fig. 4 when $E_b/N_0 = 10$ dB. Although the perfect equalization has not been guaranteed theoretically, we can expect from the results that the perfect equalization can be achieved by the DCMA.

## 5. Conclusion

The application of the DCMA to a blind adaptive equalizer for AR channels has been considered. The DCMA-based anchored FIR equalizer has been shown, for a first-order AR channel, to completely suppress ISI regardless of the gain along the direct path and the initial weights. The results of simulation have shown that the DCMA equalizer outperforms conventional equalizers.

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