LETTER

Distributed Mutually Referenced Equalization

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SUMMARY We propose a distributed blind equalization method for wireless sensor networks, in which a source sends data and each node performs time-domain equalization to estimate the data from a received signal that is affected by inter-symbol interference. The equalization can be performed distributively based on the mutually referenced equalization principle. Even if the nodes in the network are not fully connected to each other, the average consensus technique enables us to perform the equalization of all channels.

key words: *blind equalization, average consensus, wireless sensor network*

1. Introduction

Wireless sensor networks utilizing both network technology and sensing technology have been actively studied. They consist of many distributed sensor nodes and perform various types of data processing by exchanging information between the nodes. Distributed adaptive signal processing, which is one such type of data processing where the sensor nodes adaptively adjust their filter coefficients by cooperating with each other, has attracted much attention [1].

One application of distributed adaptive signal processing is distributed equalization. When a signal transmitted from a source passes through frequency-selective channels and is received by the sensor nodes, each node with adaptive filters performs time-domain equalization to suppress inter-symbol interference (ISI) [2], [3].

A blind adaptive algorithm is an attractive approach because pilot symbols are not required. Various blind adaptive equalization algorithms have been proposed [2]–[4]. In the methods reported in previous studies, however, nodes need to exchange many continuous-valued signals among each other, such as their received signals, their filter coefficients, and estimated channels. Exchanging such a large amount of signals increases the computation cost and number of data transmissions.

Recently, the distributed generalized Sato algorithm (d-GSA) was proposed [5], in which each node sends only a scalar signal to the other nodes. It can significantly reduce the cost of computation and communication. However, this algorithm requires that all the channels from the source to the nodes are almost the same. Thus, in a practical situation where the channels are sufficiently different from each other, we cannot employ d-GSA.

The present letter proposes a distributed blind equalization method based on the mutually referenced equalization principle [6]. Unlike the previous study [5], we assume that the channels from the source to the nodes are totally different from each other. In our method, each node sends only two scalar signals to the other nodes. We show that, by using the average consensus technique [7], equalization can be performed even if wireless links are established among only some of the nodes.

2. Distributed Equalization

2.1 Wireless Sensor Network Model

Let us consider a situation involving K distributed sensor nodes and one source, which transmits a data symbol (see Fig. 1). The data from the source passes through frequency-selective channels and is received by K nodes. Each sensor node performs time-domain equalization to estimate the data from the received signal, which is affected by ISI, while exchanging information only with the adjacency nodes.

The structure of the kth node is shown in Fig. 1. A data symbol s_n from the source at time n passes through the frequency-selective channels, and the kth node receives the signal $y_{k,n}$, which is expressed by

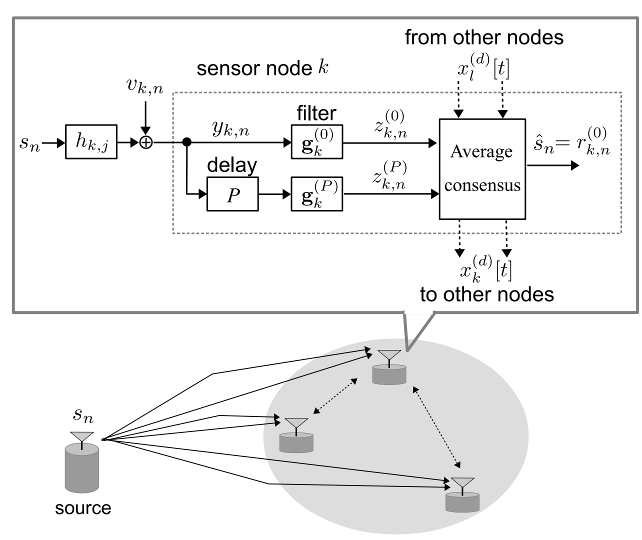

Fig. 1 Wireless sensor network.

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$$y_{k,n} = \sum_{j=0}^{L} h_{k,j} s_{n-j} + v_{k,n}, \quad k = 1, 2, \cdots, K, \tag{1}$$

where $h_{k,j}$ is the impulse response of the channel from the source to the kth node, L is the order of the channel impulse responses, and $v_{k,n}$ is additive white Gaussian noise.

In the kth node, the received signal $y_{k,n}$ is delayed and passes through two adaptive FIR filters with length M. The output of the FIR filter with delay $d \in \{0, P\}$ is given by

$$z_{k,n}^{(d)} = \sum_{j=0}^{M-1} g_{k,j}^{(d)*} y_{k,n+d-j} = \mathbf{g}_k^{(d)\mathrm{H}} \mathbf{y}_{k,n+d}, \quad d = 0, P, \tag{2}$$

where $g_{k,j}^{(d)}$ is a filter coefficient of the FIR filter,

$$\mathbf{g}_k^{(d)} = \begin{bmatrix} g_{k,0}^{(d)} & \cdots & g_{k,M-1}^{(d)} \end{bmatrix}^{\mathrm{T}} \in \mathbb{C}^{M\times 1},$$
$$\mathbf{y}_{k,n+d} = \begin{bmatrix} y_{k,n+d} & \cdots & y_{k,n+d-M+1} \end{bmatrix}^{\mathrm{T}} \in \mathbb{C}^{M\times 1}.$$

Here, the superscripts T and H denote the transpose and conjugate transpose of a vector, respectively.

For data symbol estimation and filter coefficients update, each node requires the averaged output signals of all the nodes for the filter with delay d, which is given by

$$r_n^{(d)} = \frac{1}{K}\left(z_{1,n}^{(d)} + \cdots + z_{K,n}^{(d)}\right), \ d = 0, P. \tag{3}$$

However, each node cannot obtain all the outputs $z_{k,n}^{(d)}$ because each node connects only with its neighbor nodes. Thus, each node estimates $r_n^{(d)}$ according to the average consensus technique [7] as will be described in Sect. 2.2. Let $r_{k,n}^{(d)}$ be the estimation of $r_n^{(d)}$ in the kth node. Then, $r_{k,n}^{(d)}$ for $d = 0, P$ are used to update filter coefficients as will be described in Sect. 2.3 and to estimate the data symbol as $\hat{s}_n = r_{k,n}^{(0)}$.

2.2 Averaged Output Signal Estimation by Average Consensus

To obtain the estimate $r_{k,n}^{(d)}$ of the averaged output signal $r_n^{(d)}$, we employ the average consensus technique [7]. The kth node has two auxiliary variables $x_k^{(d)}[t]$ ($d = 0, P$) and exchanges $x_k^{(d)}[t]$ between the adjacency nodes connecting with the kth node through wireless links (see Fig. 1). We assume that the communication channels between nodes are perfectly noiseless and distortionless. The value of $x_k^{(d)}[t]$ is updated by the following equation:

$$x_k^{(d)}[t+1] = x_k^{(d)}[t] + \sum_{l\in\mathcal{N}_k} W_{k,l}\left(x_l^{(d)}[t] - x_k^{(d)}[t]\right), \tag{4}$$

where $\mathcal{N}_k$ is the set of nodes connected with the kth node and $W_{k,l}$ is the (k, l) element of the weight matrix $\mathbf{W}$. If $\mathbf{W}$ satisfies some conditions, $x_k^{(d)}[t]$ converges to the average of the initial values of all the nodes as $t \to \infty$, i.e., $\lim_{t\to\infty} x_k^{(d)}[t] = (1/K)\sum_{k=1}^{K} x_k^{(d)}[0]$, $\forall k$ [7]. Thus, by setting the initial value as $x_k^{(d)}[0] = z_{k,n}^{(d)}$, $x_k^{(d)}[t]$ converges to $r_n^{(d)}$ as $t \to \infty$. Since the number of iterations is finite in a practical situation, the updated value after R times iterations is used as the estimated averaged output signal, i.e., $r_{k,n}^{(d)} = x_k^{(d)}[R]$.

2.3 Distributed Equalization Algorithm

Each node distributively adjusts the filter coefficients based on the mutually referenced equalization principle [6] such that the following cost function is minimized:

$$J(\mathbf{g}) = \mathrm{E}\left[\left|r_n^{(0)} - r_n^{(P)}\right|^2\right], \tag{5}$$

where $\mathbf{g} = \left[\mathbf{g}_1^{\mathrm{T}} \cdots \mathbf{g}_K^{\mathrm{T}}\right]^{\mathrm{T}}$ and $\mathbf{g}_k = \begin{bmatrix} \mathbf{g}_k^{(0)\mathrm{T}} & \mathbf{g}_k^{(P)\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$. To minimize the cost function, each node updates its filter coefficients by using the stochastic gradient algorithm as follows:

$$\begin{aligned} \mathbf{g}_k[n+1] &= \mathbf{g}_k[n] - \mu \frac{\partial \left|r_{k,n}^{(0)} - r_{k,n}^{(P)}\right|^2}{\partial \mathbf{g}_k^*} \\ &= \mathbf{g}_k[n] - \mu \frac{1}{K} \tilde{\mathbf{y}}_{k,n}\left(r_{k,n}^{(0)*} - r_{k,n}^{(P)*}\right), \end{aligned} \tag{6}$$

where $\mu > 0$ is a step gain and $\tilde{\mathbf{y}}_{k,n} = \begin{bmatrix} \mathbf{y}_{k,n}^{\mathrm{T}} & -\mathbf{y}_{k,n+P}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$. Note that the estimated averaged signal $r_{k,n}^{(d)}$ is used for the update instead of $r_n^{(d)}$. Moreover, the norm constraint $||\mathbf{g}_k|| = 1$ is imposed to avoid a trivial solution $\mathbf{g}_k = \mathbf{0}$. Owing to this constraint, $\mathbf{g}_k$ is normalized as follows:

$$\mathbf{g}_k[n+1] = \frac{\mathbf{g}_k[n+1]}{||\mathbf{g}_k[n+1]||}. \tag{7}$$

The proposed algorithm is summarized as follows:

Step 1 $n = 0$, and initialize $\mathbf{g}_k[0]$, and determine the step gain μ, the number of iterations R for average consensus, and the number of iterations I for equalization.
Step 2 Obtain the filter output $z_{k,n}^{(d)}$ by Eq. (2).
Step 3 Set $x_k^{(d)}[0] = z_{k,n}^{(d)}$, update $x_k^{(d)}[t]$ R times by using Eq. (4), and set $r_{k,n}^{(d)} = x_k^{(d)}[R]$. Obtain $\hat{s}_n = r_{k,n}^{(0)}$.
Step 4 Update the filter coefficient according to Eqs. (6) and (7).
Step 5 $n = n + 1$, and return to **Step 2** until n reaches I.

3. Simulation Results

Unless otherwise noted, we used the simulation parameters in Table 1. The channel coefficients are modeled as complex Gaussian random variables with zero mean and variances $\rho_l^2 = \lambda\exp(-\alpha l), l = 0, \cdots, L$, where $\alpha = 0.23$ and λ denotes a constant ensuring the unit average energy of the channel. The received SNR is defined as $\mathrm{SNR} = K\frac{\sigma_s^2 p_0}{\sigma_n^2}$, where σ_s^2 is the transmitted signal power, σ_n^2 is the noise

Table 1 Simulation parameters.

Modulation scheme	coherent QPSK
Channel order L	2
Filter length M	4
Number of nodes K	5
Number of iterations for average consensus R	15
Number of iterations for equalization I	10^3
Delay P	5

variance, and p_0 is the sum of the mean square of all paths for each channel. We assume that phase ambiguity, which is inherent to all blind schemes, can be resolved. In practice, the ambiguity should be resolved for coherent demodulation by transmitting short training symbols or adopting differential encoding/decoding. BER is obtained by averaging over 10^3 simulation trials where each trial has different channels. BER in each trial is measured over 10^4 QPSK symbols after I iterations of updating the filter coefficients.

In this section, we assume that each node in the sensor network connects with only two neighboring nodes. For the update of Eq. (4), we employ Metropolis-Hasting weights [7]:

$$W_{k,l} = \begin{cases} \frac{1}{1+\max\{d_k, d_l\}} & \text{if } k \text{ and } l \text{ are connected} \\ 1 - \sum_{k \neq l} W_{k,l} & \text{if } l = k \\ 0 & \text{otherwise} \end{cases},$$

where d_k is the number of nodes connecting with the kth node.

Figure 2 shows the BER performance of the proposed algorithm (labeled as "distributed MRE"). For each SNR, we chose the step gain from $\mu \in \{10^{-3}, 10^{-2}, 10^{-1}\}$ such that the lowest BER is achieved. We also show the results of two centralized equalizations (centralized MRE ($||\mathbf{g}|| = 1$) [6] and centralized MRE ($||\mathbf{g}_k|| = 1$)): the former determines all the filter coefficients $\mathbf{g}$ in a centralized manner such that the cost function (5) is minimized under the norm constraint $||\mathbf{g}|| = 1$ by a batch method using 10^3 received samples; the latter updates the filter coefficients of each node according to Eqs. (6) and (7) on the assumption that the outputs of all the nodes are known (i.e., $r^{(d)}_{k,n} = r^{(d)}_n$ in Eq. (6)). In addition, we show the result of the distributed generalized Sato algorithm (d-GSA) [5]. We can see the effectiveness of the proposed algorithm in the result. On the other hand, the performance of d-GSA is very poor because d-GSA is expected to work well only in an environment where all the channels from the source to the nodes are almost the same. Furthermore, the performance of distributed MRE is approximately 3dB better than that of centralized MRE ($||\mathbf{g}|| = 1$). Both centralized MRE ($||\mathbf{g}|| = 1$) and distributed MRE have the same cost function (5) but different constraints $||\mathbf{g}|| = 1$ and $||\mathbf{g}_k|| = 1$, respectively. Because of this difference, distributed MRE could prevent the filter outputs of some nodes from being weakened. Moreover, the performance of distributed MRE and centralized MRE ($||\mathbf{g}_k|| = 1$) is almost same. This result indicates that the constraint $||\mathbf{g}_k|| = 1$ is better than $||\mathbf{g}|| = 1$, and that the average consensus for distributed MRE works well. As can be seen in Fig. 2, the diversity order of MREs is about 1.5 despite using five antennas. The reduction of diversity order is a disadvantage of MRE.

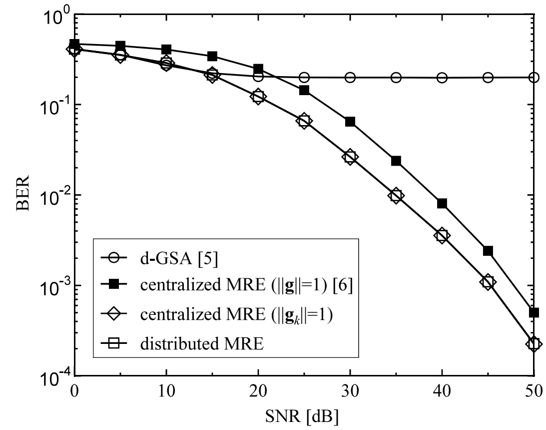

Fig. 2 Comparison of d-GSA and distributed MRE.

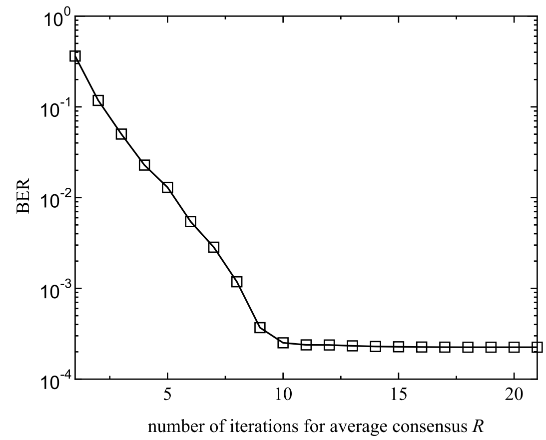

Fig. 3 Influence of the number of iterations R on the average consensus (SNR = 50dB, $\mu = 10^{-3}$).

Figure 3 shows the influence of the number of iterations for average consensus R, which corresponds to the number of information exchanges between nodes for one data symbol estimation. With increasing R, BER reduces and converges with $R = 10$ at most. Thus, only a small R value is sufficient to achieve consensus, i.e., to estimate a data symbol.

4. Conclusion

The present letter proposed the distributed blind equalization algorithm based on the mutually referenced equalization principle for wireless sensor networks. Numerical simulations showed that, if the channels are totally different from each other, the performance of the proposed algorithm is significantly better than that of d-GSA and slightly better than that of centralized MRE. Even if each node connects with only some of the nodes in a network, we can equalize the channels by using the average consensus technique. To overcome the disadvantage of MRE, developing equalization schemes with higher diversity order is worthwhile.

References

[1] A. Sayed and C. Lopes, "Adaptive processing over distributed networks," IEICE Trans. Fundamentals, vol.E90-A, no.8, pp.1504–1510, Aug. 2007.
[2] R. Abdolee and B. Champagne, "Distributed blind adaptive algorithms based on constant modulus for wireless sensor networks," Proc. IEEE Int. Conf. Wireless Mobile Commun., pp.303–308, Sept. 2010.
[3] C. Yu and L. Xie, "On recursive blind equalization in sensor networks," IEEE Trans. Signal Process., vol.63, no.3, pp.662–672, Feb. 2015.
[4] A. Bertrand and M. Moonen, "Distributed LCMV beamforming in a wireless sensor network with single-channel per-node signal transmission," IEEE Trans. Signal Process., vol.61, no.13, pp.3447–3459, July 2013.
[5] Y. Liu and Y. Cai, "Distributed blind equalization in networked systems," Proc. IEEE Int. Conf. Acoust. Speech Signal Process., pp.4296–4300, March 2017.
[6] G.B. Giannakis and C. Tepedelenlioğlu, "Direct blind equalizers of multiple FIR channels: A deterministic approach," IEEE Trans. Signal Process., vol.47, no.1, pp.62–74, Jan. 1999.
[7] L. Xiao, S. Boyd, and S.J. Kim, "Distributed average consensus with least-mean-square deviation," J. Parallel Distr. Comput., vol.67, no.1, pp.33–46, Jan. 2007.