PAPER

Design Methods of Filter-and-Forward Relay Beamforming for OFDM-Based Cognitive Networks

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SUMMARY In this paper, we propose filter-and-forward beamforming (FF-BF) for cognitive two-way relay networks in which secondary users employ an orthogonal frequency-division multiplexing (OFDM) system. Secondary transceivers communicate with each other through multiple relays to obtain BF gain as well as to suppress the interference between the primary and secondary users who share the same spectrum. We consider two FF-BF design methods to optimize the relay filter. The first method enhances the quality of service of the secondary network by maximizing the worst subcarrier signal-to-interference-plus-noise ratio (SINR) subject to transmit power constraints. The second method suppresses the interference from the secondary network to the primary network through the minimization of the relay transmission power subject to subcarrier SINR constraints. Simulation results show that the proposed FF-BF improves system performance in comparison to amplify-and-forward relay BF.
key words: *cognitive radio, filter-and-forward, relay beamforming, OFDM*

1. Introduction

Cognitive radio (CR) addresses the dual problems of underutilization of the licensed spectrum and dramatically growing demand for wireless radio spectrum [1]. Specifically, in the CR underlay paradigm, unlicensed users (secondary user (SU)) are allowed to transmit simultaneously with licensed users (primary user (PU)) who share the same frequency as long as the interference from SU to PU is limited to a certain level that ensures that the interference does not degrade PU's communication [2]. Consequently, the SU in a CR network is always forced to operate with low transmit power to limit the interference to the PU, and as a result, the SU suffers degradation in the quality of service (QoS). A smart solution to this problem is a cooperative relaying technology in which SUs communicate with each other by using multiple spatially-separated relays.

In non-CR networks, distributed relay beamforming (BF) [3] has recently gained much interest among cooperative relaying techniques since it can provide BF gain even for single antenna users. Many works on relay BF focus on amplify-and-forward (AF) relaying over frequency-flat channels [4]. Recently, another simple relaying approach, namely filter-and-forward (FF) relaying, which can be regarded as an extension of the AF relaying, has been investigated to combat inter-symbol interference (ISI) in frequency-selective channels [5], [6]. For FF-BF two-way relay networks, the total relay transmitted power minimization approach [5] and the minimum receiver SINR maximization approach [6] have been developed. In these works, FF relays are responsible for time-domain equalization in single-carrier systems without both cyclic prefix (CP) and frequency-domain equalizer (FDE). Whereas in OFDM systems, the necessity of time-domain equalization can be eliminated by using CP and FDE so that an FF relay can be devoted to enhancing the system performance as studied in [7]. However, the authors in [7] considered only a single-relay network, while FF-BF for OFDM systems has not been considered.

On the other hand, in CR networks, several relay BF designs have been developed. An interference minimization approach for AF-BF with full channel state information (CSI) has been introduced in [8]. Moreover, an SINR balancing and interference minimization approach for AF-BF with partial CSI has been presented in [9], [10]. Furthermore, an FF-BF approach which minimizes the peak interference power from the secondary network subject to the QoS requirements has been introduced in [11]. However, [8]–[11] have been developed for single-carrier based CR networks, while to our best knowledge, no literature on FF-BF approach for OFDM-based CR networks exists.

In this paper, we consider FF-BF two-way relaying for OFDM-based CR networks. The contributions of this paper include:

- Unlike [8]–[11], OFDM-based CR networks are considered. Since ISI is no longer a problem, the design of FF-BF can be devoted to improving the system performance as well as to limiting both the interference to PU and the interference from PU.
- We propose two FF-BF design methods: the first method maximizes the worst subcarrier SINR under a relay transmission power constraint; the second one minimizes the relay transmission power while subcarrier SINRs are kept at a sufficiently high level. Both methods can be applied even if partial CSI is available.
- Simulation results demonstrate that the proposed FF-BF is superior to AF-BF, and the performance of the proposed FF-BF improves with the number of relays.

The rest of this paper is organized as follows. Section 2 describes the system model. Section 3 studies the FF-BF

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design problems. Section 4 evaluates the performance of the proposed design methods, followed by the conclusion in Sect. 5.

Throughout the paper, we use the following notations: $(\cdot)^T$, $(\cdot)^H$ and $\otimes$ denote the transpose, Hermitian transpose and Kronecker product, respectively. $\mathrm{diag}(\mathbf{A}_1 \ \cdots \ \mathbf{A}_n)$ is a block diagonal matrix with $\mathbf{A}_i$ on diagonal. $\mathrm{tr}(\mathbf{A})$ indicates the trace of $\mathbf{A}$. $\mathbf{A} \geq 0$ implies that $\mathbf{A}$ is a semidefinite matrix. $\mathrm{Toeplitz}(\mathbf{A}, M, l)$ denotes an $NM\times(L+M-1)l$ block Toeplitz matrix whose first block row is an $N \times lL$ matrix $\mathbf{A}$ with $N \times (M-1)l$ trailing zeros and mth block row is its shifted matrix with a shift offset of $(m-1)l$. $\mathrm{vec}(\mathbf{A})$ denotes the vector obtained by stacking the columns of $\mathbf{A}$ and $\mathrm{mat}(\mathbf{A})$ is the inverse of $\mathrm{vec}(\mathbf{A})$.

2. System Model

We consider a cognitive radio system in which SUs transmit simultaneously with a PU who shares the same frequency band as shown in Fig. 1. In the secondary network, there are two SU transceivers denoted by $SU_i (i = 1, 2)$. SU_is communicate with each other through M relays. All of the SU transceivers and relays operate in a half-duplex mode. During the first phase, SUs transmit their data to the relays and the relays perform FIR filtering on the received signals which include both signals from SU_i and a signal from a primary transmitter (PT). During the second phase, relays broadcast the filtered signal to SU_is. In the primary network, the PT communicates with the corresponding primary receiver (PR), regardless of the secondary network. Each transmitter and receiver in this system equips with a single antenna.

We make the following assumptions. There are no direct links between SU_i and PT/PR [8], [11]. SUs are allowed to communicate with each other only when SUs are

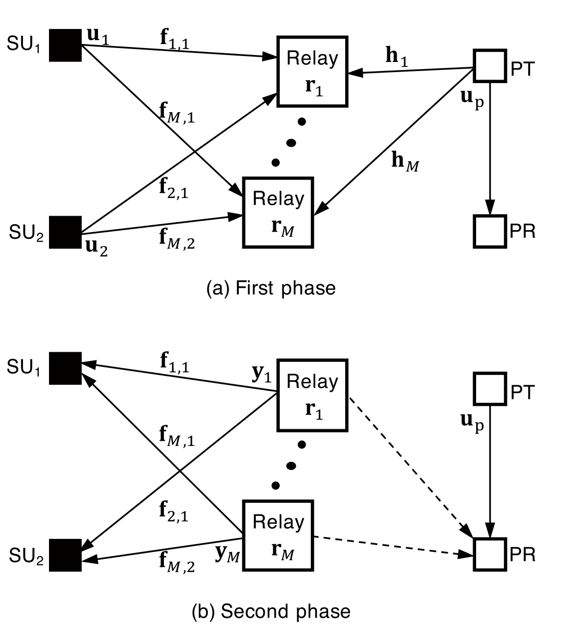


Fig. 1 System model of a cognitive FF-BF two-way relay network.

not close to PT/PR since unprocessed direct interference may seriously degrade the performance of PU. All the channels are frequency-selective channels which can be modeled as FIR filters. To demodulate properly, the impulse response length of the overall channel of SU_1-relays-SU_2 is less than that of the CP. The channel between SU_i and a relay is constant at least during the consecutive first and second phases. Its full CSI can be known at the relay by using orthogonal pilots. Then the CSI obtained at each relay is sent to a base station where the filter coefficients are computed and are fed back to each relay. Meanwhile, the full CSI of the PT-relay channels is difficult to obtain unless the secondary network has enough knowledge about the transmit signal of PT. Thus, it is reasonable to assume that only the second-order statistics (SOS) of the PT-relay channels is available at the relays. Moreover, the relay-PR channels are unknown at the relays.

In the first phase, at the SU_i transmitter, an OFDM signal is transmitted to relays. Let us consider the transmission of the data block of length N represented by $\mathbf{s}_i = [s_i[0] \ s_i[1] \ \cdots \ s_i[N-1]]^T$ where each data symbol $s_i[k]$ is assumed to be a zero-mean independent random variable with variance P_s. After normalized inverse discrete Fourier transform (IDFT) is applied, the time-domain signal vector $\mathbf{x}_i$ is given by

$$\mathbf{x}_i = [x_i[0] \ x_i[1] \ \cdots \ x_i[N-1]]^T = \mathbf{Q}_N \mathbf{s}_i \tag{1}$$

where the IDFT matrix $\mathbf{Q}_N$ whose (k,l)th entry is $\frac{1}{\sqrt{N}} e^{j(2\pi/N)(k-1)(l-1)}$ has a unitary property $\mathbf{Q}_N \mathbf{Q}_N^H = \mathbf{I}_N$. The CP of length L_p is added to the signal vector $\mathbf{x}_i$ to form the transmitted block whose kth entry is given by

$$u_i[k] = x_i[\mathrm{mod}(k + N - L_p, N)] \tag{2}$$

for $k = 0, 1, \cdots, N + L_p - 1$, where $\mathrm{mod}(m, n)$ is the remainder of m modulo n. The power of the transmitted signal $u_i[k]$ of SU_i becomes $E[|u_i[k]|^2] = P_s$ according to the property of IDFT.

At the mth relay, the time-domain received baseband signal at time n is given by

$$\begin{aligned} v_m[n] = & \sum_{i=1}^{2} \sum_{l=0}^{L-1} f_{m,i}[l] u_i[n-l] + \sum_{l=0}^{L-1} h_m[l] u_{\mathrm{p}}[n-l] \\ & + \eta_m[n] \end{aligned} \tag{3}$$

where $f_{m,i}[l]$, $h_m[l]$ are the channel impulse response between SU_i and the mth relay, and that between PT and the mth relay, respectively, L is the length of channels, $u_{\mathrm{p}}[n]$ is the signal transmitted by PT and $\eta_m[n] \sim \mathcal{CN}(0, \sigma_r^2)$ denotes spatially and temporally uncorrelated additive white Gaussian noise (AWGN). The mth relay obtains the FIR filtered time-domain signal which can be given by

$$y_m[n] = \sum_{l=0}^{L_r-1} r_m[l] v_m[n-l] \tag{4}$$

where $r_m[l]$ is the filter coefficients at the mth relay and L_r

is the length of the filter. For simple presentation, we write the transmitted signal at the mth relay in a vector form as

$$\mathbf{y}_m = \sum_{i=1}^{2} \mathbf{F}_{m,i}\tilde{\mathbf{R}}_m\mathbf{u}_i + \mathbf{H}_m\tilde{\mathbf{R}}_m\mathbf{u}_\mathrm{p} + \mathbf{R}_m\boldsymbol{\eta}_m \tag{5}$$

where

$$\begin{aligned}
\mathbf{y}_m &= [y_m[-L+1] \cdots y_m[N-1]]^T,\\
\mathbf{u}_i &= [x_i[-2L-L_r+3] \cdots x_i[0] \cdots x_i[N-1]]^T,\\
\mathbf{u}_\mathrm{p} &= [u_\mathrm{p}[-2L-L_r+3] \cdots u_\mathrm{p}[0] \cdots u_\mathrm{p}[N-1]]^T,\\
\boldsymbol{\eta}_m &= [\eta_m[-L-L_r+2] \cdots \eta_m[N-1]]^T,\\
\mathbf{F}_{m,i} &= \mathrm{Toeplitz}(\mathbf{f}_{m,i}^T, N+L-1, 1),\\
\mathbf{H}_m &= \mathrm{Toeplitz}(\mathbf{h}_m^T, N+L-1, 1),\\
\mathbf{R}_m &= \mathrm{Toeplitz}(\mathbf{r}_m^T, N+L-1, 1),\\
\tilde{\mathbf{R}}_m &= \mathrm{Toeplitz}(\mathbf{r}_m^T, N+2L-2, 1),\\
\mathbf{r}_m &= [r_m[L_r-1]\; r_m[L_r-2] \cdots r_m[0]]^T,\\
\mathbf{f}_{m,i} &= [f_{m,i}[L-1]\; f_{m,i}[L-2] \cdots f_{m,i}[0]]^T,\\
\mathbf{h}_m &= [h_m[L-1]\; h_m[L-2] \cdots h_m[0]]^T.
\end{aligned} \tag{6}$$

In the second phase, the mth relay broadcasts its filtered signal $y_m[n]$. At the SU_i receiver, the received time-domain signal is given by

$$t_i[T_1+n] = \sum_{m=1}^{M}\sum_{l=0}^{L-1} f_{m,i}[l]y_m[n-l] + n_i[T_1+n] \tag{7}$$

where T_1 is the first phase duration and $n_i[n] \sim \mathcal{CN}(0,\sigma_i^2)$ denotes spatially and temporally uncorrelated AWGN at the SU_i. By a vector form, the received signal after CP removal can be represented by

$$\mathbf{t}_i = \sum_{m=1}^{M} \hat{\mathbf{F}}_{m,i}\mathbf{y}_m + \mathbf{n}_i \tag{8}$$

where $\mathbf{t}_i = [t_i[T_1]\; t_i[T_1+1] \cdots t_i[T_1+N-1]]$, $\hat{\mathbf{F}}_{m,i} = \mathrm{Toeplitz}(\mathbf{f}_{m,i}^T, N, 1)$ and $\mathbf{n}_i = [n_i[T_1] \cdots n_i[T_1+N-1]]^T$. The received signal $\mathbf{t}_i$ of SU_i contains a self-signal component coming via relays. Since the full CSI in the secondary network is assumed to be available at SU_i, we can subtract the self signal from $t_i[n]$. After that, DFT is applied to obtain a frequency-domain signal vector $\mathbf{z}_i$ of SU_i as

$$\begin{aligned}
\mathbf{z}_i &= \mathbf{Q}_N^H\left(\mathbf{t}_i - \sum_{m=1}^{M}\hat{\mathbf{F}}_{m,i}\mathbf{F}_{m,i}\tilde{\mathbf{R}}_m\mathbf{u}_i\right)\\
&= \mathbf{Q}_N^H\sum_{m=1}^{M}\Big(\underbrace{\hat{\mathbf{F}}_{m,i}\,\mathbf{F}_{m,j}\tilde{\mathbf{R}}_m\mathbf{u}_j}_{\text{desired component}} + \underbrace{\hat{\mathbf{H}}_m\mathbf{F}_{m,i}\tilde{\mathbf{R}}_m\mathbf{u}_\mathrm{p}}_{\text{interference component}}\\
&\quad + \underbrace{\hat{\mathbf{F}}_{m,i}\mathbf{R}_m\boldsymbol{\eta}_m\Big) + \mathbf{Q}_N^H\mathbf{n}_i}_{\text{noise component}}, \quad i,j=1,2 \text{ and } i\neq j
\end{aligned} \tag{9}$$

where $\hat{\mathbf{H}}_m = \mathrm{Toeplitz}(\mathbf{h}_m^T, N, 1)$ and $\mathbf{z}_i = [z_i[0] \cdots z_i[N-1]]^T$. Finally, by performing frequency-domain equalization on $\mathbf{z}_i$, the estimation of data $\mathbf{s}_i$ is obtained.

3. FF-BF Design as Optimization Problem

The purpose of FF-BF design is to determine the relay filter coefficients vector $\mathbf{r} = [\mathbf{r}_1^T \cdots \mathbf{r}_M^T]^T$. Inspired by the ideas in the previous works [5]–[11], we formulate the FF-BF design as optimization problems concerning the SINR of the received signal and the relay transmit power. Thus, in the first part of this section, our efforts focus on expressing the SINR in terms of $\mathbf{r}$. More specifically:

- The power of the desired and noise components in (9) are expressed as a quadratic form of $\mathbf{r}$.
- To obtain a quadratic form for the interference power, we derive the correlation matrix of the PT-relay channels, and also derive the correlation matrix of the PT transmission signal for two types of signaling.
- The total relay transmission power is examined as well.

In the second part, we propose two FF-BF relay design methods formulated as constrained optimization problems, which are represented using these quadratic forms.

3.1 Quadratic Form of Each Component's Power

Let us consider the kth subcarrier DFT output of SU_i. The kth entry of $\mathbf{z}_i$ in (9) is given by

$$\begin{aligned}
z_i[k] = \sum_{m=1}^{M}\Big(&\mathbf{q}_k^H\hat{\mathbf{F}}_{m,i}\mathbf{F}_{m,j}\tilde{\mathbf{R}}_m\mathbf{u}_j + \mathbf{q}_k^H\hat{\mathbf{H}}_m\mathbf{F}_{m,i}\tilde{\mathbf{R}}_m\mathbf{u}_\mathrm{p}\\
&+\mathbf{q}_k^H\hat{\mathbf{F}}_m\mathbf{R}_m\boldsymbol{\eta}_m\Big) + \mathbf{q}_k^H\mathbf{n}_i, \quad i,j=1,2 \text{ and } i\neq j
\end{aligned} \tag{10}$$

for $k=0,1,\cdots,N-1$, where $\mathbf{q}_k^H$ is the $(k+1)$th row of $\mathbf{Q}_N^H$. In (10), the desired, interference from PT to SU_i, and noise components are given by

$$z_{S,i}[k] = \sum_{m=1}^{M}\mathbf{q}_k^H\hat{\mathbf{F}}_{m,i}\mathbf{F}_{m,j}\tilde{\mathbf{R}}_m\mathbf{u}_j = \mathbf{q}_k^H\hat{\mathbf{F}}_i\mathbf{F}_j\tilde{\mathbf{R}}\mathbf{u}_j, \tag{11}$$

$$z_{I,i}[k] = \sum_{m=1}^{M}\mathbf{q}_k^H\hat{\mathbf{H}}_m\mathbf{F}_{m,i}\tilde{\mathbf{R}}_m\mathbf{u}_\mathrm{p} = \mathbf{q}_k^H\hat{\mathbf{H}}\mathbf{F}_i\tilde{\mathbf{R}}\mathbf{u}_\mathrm{p}, \tag{12}$$

$$z_{N,i}[k] = \sum_{m=1}^{M}\mathbf{q}_k^H\hat{\mathbf{F}}_{m,i}\mathbf{R}_m\boldsymbol{\eta}_m + \mathbf{q}_k^H\mathbf{n}_i \tag{13}$$

where $\hat{\mathbf{F}}_i = [\hat{\mathbf{F}}_{1,i} \cdots \hat{\mathbf{F}}_{M,i}]$, $\hat{\mathbf{H}} = [\hat{\mathbf{H}}_1 \cdots \hat{\mathbf{H}}_M]$, $\mathbf{F}_i = \mathrm{diag}([\mathbf{F}_{1,i} \cdots \mathbf{F}_{M,i}])$, and $\tilde{\mathbf{R}} = [\tilde{\mathbf{R}}_1^T \cdots \tilde{\mathbf{R}}_M^T]^T$. In the following, we derive the power of these components.

3.1.1 Desired and Noise Component

The power of the desired component $z_{S,i}[k]$ in (11) can be expressed by a quandratic form as[†]

[†] $\mathrm{tr}(\mathbf{ABC}) = \mathrm{tr}(\mathbf{BCA})$ is used to derive the third line and $\mathrm{tr}(\mathbf{A}^T\mathbf{BCD}^T) = \mathrm{vec}(\mathbf{A})^T(\mathbf{D}\otimes\mathbf{B})\mathrm{vec}(\mathbf{C})$ is used to derive the fourth line.

$$\begin{aligned} P_{S,k} &= E[|z_{S,i}[k]|^2] \\ &= \mathrm{tr}\left(E[\mathbf{u}_j^H \tilde{\mathbf{R}}^H \mathbf{F}_j^H \hat{\mathbf{F}}_i^H \mathbf{q}_k \mathbf{q}_k^H \hat{\mathbf{F}}_i \mathbf{F}_j \tilde{\mathbf{R}} \mathbf{u}_j]\right) \\ &= \mathrm{tr}\left(\tilde{\mathbf{R}}^H \mathbf{F}_j^H \hat{\mathbf{F}}_i^H \mathbf{Q}^{(k)} \hat{\mathbf{F}}_i \mathbf{F}_j \tilde{\mathbf{R}} E[\mathbf{u}_j \mathbf{u}_j^H]\right) \\ &= \mathrm{vec}(\tilde{\mathbf{R}})^H \left\{\mathbf{\Omega}_j^T \otimes (\mathbf{F}_j^H \hat{\mathbf{F}}_i^H \mathbf{Q}^{(k)} \hat{\mathbf{F}}_i \mathbf{F}_j)\right\} \mathrm{vec}(\tilde{\mathbf{R}}) \\ &= \mathbf{r}^H \mathbf{E} \left\{\mathbf{\Omega}_j^T \otimes (\mathbf{F}_j^H \hat{\mathbf{F}}_i^H \mathbf{Q}^{(k)} \hat{\mathbf{F}}_i \mathbf{F}_j)\right\} \mathbf{E}^H \mathbf{r} \\ &= \mathbf{r}^H \mathbf{\Phi}_{S,k} \mathbf{r} \end{aligned} \tag{14}$$

where $\mathbf{Q}^{(k)} = \mathbf{q}_k \mathbf{q}_k^H$, $\mathbf{\Omega}_j = E[\mathbf{u}_j \mathbf{u}_j^H]$, $\mathbf{E}^H \mathbf{r} = \mathrm{vec}(\tilde{\mathbf{R}})$ and

$$\mathbf{\Phi}_{S,k} = \mathbf{E} \left\{\mathbf{\Omega}_j^T \otimes (\mathbf{F}_j^H \hat{\mathbf{F}}_i^H \mathbf{Q}^{(k)} \hat{\mathbf{F}}_i \mathbf{F}_j)\right\} \mathbf{E}^H. \tag{15}$$

For the transmitted OFDM signal $\mathbf{u}_j$ described in the previous section, its correlation matrix becomes

$$\mathbf{\Omega}_j = P_s \left[\begin{array}{c|c} \mathbf{I}_{L_p} & \mathbf{0}_{L_p \times (N-L_p)} \ \ \mathbf{I}_{L_p} \\ \hline \begin{matrix} \mathbf{0}_{(N-L_p)\times L_p} \\ \mathbf{I}_{L_p} \end{matrix} & \mathbf{I}_{L_N} \end{array}\right], \quad j = 1, 2 \tag{16}$$

where $L_p = L_r + 2L - 3$ is the CP length. The matrix $\mathbf{E}$ is introduced to separate $\mathbf{r}$ from $\mathrm{vec}(\tilde{\mathbf{R}})$ and is given by

$$\begin{aligned} \mathbf{E} &= \mathrm{Toeplitz}(\mathbf{E}_b, M, N+2L-2) \\ &\in \mathbb{R}^{ML_r \times M(N+2L-2)(N+L_p)}, \\ \mathbf{E}_b &= \mathrm{Toeplitz}(\mathbf{E}_{bb}, L_r, M(N+2L-2)) \\ &\in \mathbb{R}^{L_r \times \left(M(N+2L-2)(N+L_p) - (M-1)(N+2L-2)\right)}, \\ \mathbf{E}_{bb} &= [\tilde{\mathbf{e}}_1^T \ \ \tilde{\mathbf{e}}_2^T \ \ \cdots \ \ \tilde{\mathbf{e}}_{N+2L-3}^T \ \ \mathbf{e}_{N+2L-2}^T] \\ &\in \mathbb{R}^{1 \times \left(M(N+2L-2)^2 - (M-1)(N+2L-2)\right)} \end{aligned} \tag{17}$$

where $\tilde{\mathbf{e}}_n^T$, $\mathbf{e}_n^T$ are the nth row of $\mathbf{I}_{M(N+2L-2)}$ and $\mathbf{I}_{N+2L-2}$, respectively.

Next, we consider the kth subcarrier noise component $z_{N,i}[k]$ in (13). Due to the uncorrelated nature of noises, it holds that $E[\boldsymbol{\eta}_m \boldsymbol{\eta}_m^H] = \sigma_r^2 \mathbf{I}_{N+L+L_r-2}$ and $E[\mathbf{n}_i \mathbf{n}_i^H] = \sigma_i^2 \mathbf{I}_N$. Then, the power of the noise component can be obtained in a similar way to (14) as

$$\begin{aligned} P_{N,i,k} &= E[|z_{N,i}[k]|^2] \\ &= \sum_{m=1}^{M} \mathrm{tr}(\mathbf{R}_m^H \hat{\mathbf{F}}_{m,i}^H \mathbf{q}_k \mathbf{q}_k^H \hat{\mathbf{F}}_{m,i} \mathbf{R}_m E[\boldsymbol{\eta}_m \boldsymbol{\eta}_m^H]) + \mathbf{q}_k^H E[\mathbf{n}_i \mathbf{n}_i^H] \mathbf{q}_k \\ &= \sigma_r^2 \sum_{m=1}^{M} \mathrm{vec}(\mathbf{R}_m)^H \mathbf{M}_{m,i,k} \mathrm{vec}(\mathbf{R}_m) + \sigma_i^2 \\ &= \sigma_r^2 \sum_{m=1}^{M} \mathbf{r}_m^H \mathbf{E}_1 \mathbf{M}_{m,i,k} \mathbf{E}_1^H \mathbf{r}_m + \sigma_i^2 \\ &= \mathbf{r}^H \mathbf{\Phi}_{N,i,k} \mathbf{r} + \sigma_i^2 \end{aligned} \tag{18}$$

where

$$\begin{aligned} \mathbf{\Phi}_{N,i,k} &= \sigma_r^2 \mathrm{diag}([\mathbf{E}_1 \mathbf{M}_{1,i,k} \mathbf{E}_1^H \ \ \cdots \ \ \mathbf{E}_1 \mathbf{M}_{M,i,k} \mathbf{E}_1^H]), \\ \mathbf{M}_{m,i,k} &= \mathbf{I}_{N+L+L_r-2} \otimes (\hat{\mathbf{F}}_{m,i}^H \mathbf{q}_k \mathbf{q}_k^H \hat{\mathbf{F}}_{m,i}), \\ \mathbf{E}_1 &= \mathrm{Toeplitz}(\mathbf{E}_{1b}, L_r, N+L-1) \in \mathbb{R}^{L_r \times (N+L-1)(N+L+L_r-2)}, \end{aligned}$$

$$\mathbf{E}_{1b} = [\hat{\mathbf{e}}_1^T \ \ \hat{\mathbf{e}}_2^T \ \ \cdots \ \ \hat{\mathbf{e}}_{N+L-1}^T] \in \mathbb{R}^{1 \times (N+L-1)(N+L-1)} \tag{19}$$

where $\hat{\mathbf{e}}_n^T$ is the nth row of $\mathbf{I}_{N+L-1}$.

3.1.2 Interference Component

For the interference component from PT to SU_i $z_{I,i}[k]$ in (12), we can express its power as

$$\begin{aligned} P_{I,i,k} &= E[|z_{I,i}[k]|^2] \\ &= \mathrm{tr}\left(E[\mathbf{u}_\mathrm{p}^H \tilde{\mathbf{R}}^H \mathbf{F}_i^H \hat{\mathbf{H}}^H \mathbf{q}_k \mathbf{q}_k^H \hat{\mathbf{H}} \mathbf{F}_i \tilde{\mathbf{R}} \mathbf{u}_\mathrm{p}]\right) \\ &= \mathrm{tr}\left(\tilde{\mathbf{R}}^H \mathbf{F}_i^H E[\hat{\mathbf{H}}^H \mathbf{Q}^{(k)} \hat{\mathbf{H}}] \mathbf{F}_i \tilde{\mathbf{R}} E[\mathbf{u}_\mathrm{p} \mathbf{u}_\mathrm{p}^H]\right) \\ &= \mathrm{vec}(\tilde{\mathbf{R}})^H \left\{\mathbf{\Omega}_\mathrm{p}^T \otimes (\mathbf{F}_i^H \mathbf{\Omega}_{sos,k} \mathbf{F}_i)\right\} \mathrm{vec}(\tilde{\mathbf{R}}) \\ &= \mathbf{r}^H \mathbf{\Phi}_{I,i,k} \mathbf{r} \end{aligned} \tag{20}$$

where $\mathbf{\Omega}_{sos,k} = E[\hat{\mathbf{H}}^H \mathbf{Q}^{(k)} \hat{\mathbf{H}}]$, $\mathbf{\Omega}_\mathrm{p} = E[\mathbf{u}_\mathrm{p} \mathbf{u}_\mathrm{p}^H]$ and

$$\mathbf{\Phi}_{I,i,k} = \mathbf{E} \left\{\mathbf{\Omega}_\mathrm{p}^T \otimes (\mathbf{F}_i^H \mathbf{\Omega}_{sos,k} \mathbf{F}_i)\right\} \mathbf{E}^H. \tag{21}$$

To obtain $P_{I,i,k}$, two matrices $\mathbf{\Omega}_{sos,k}$ and $\mathbf{\Omega}_\mathrm{p}$ should be specified.

The first matrix $\mathbf{\Omega}_{sos,k}$ depends on the PT-relay channels. As mentioned before, we can access the SOS of the PT-relay channels. For this purpose, we adopt the following model for the PT-relay channels [10]

$$\mathbf{h}_m = \sqrt{\frac{1}{1+\alpha}} \bar{\mathbf{h}}_m + \sqrt{\frac{\alpha}{1+\alpha}} \breve{\mathbf{h}}_m \tag{22}$$

where $\bar{\mathbf{h}}_m$ and $\breve{\mathbf{h}}_m \sim \mathcal{CN}(0, \mathbf{I}_L)$ are the mean and variable components of the channel impulse response $\mathbf{h}_m$, respectively. The uncertainty parameter α controls the variation of $\mathbf{h}_m$ from its mean value. For a given $\bar{\mathbf{h}}_m$, we have

$$\mathbf{\Omega}_{sos,k} = \frac{1}{1+\alpha} \bar{\mathbf{H}}^H \mathbf{Q}^{(k)} \bar{\mathbf{H}} + \frac{\alpha}{1+\alpha} E[\breve{\mathbf{H}}^H \mathbf{Q}^{(k)} \breve{\mathbf{H}}] \tag{23}$$

where $\bar{\mathbf{H}} = [\bar{\mathbf{H}}_1 \ \cdots \ \bar{\mathbf{H}}_M]$, $\bar{\mathbf{H}}_m = \mathrm{Toeplitz}(\bar{\mathbf{h}}_m^T, N, 1)$, and $\breve{\mathbf{H}} = [\breve{\mathbf{H}}_1 \ \cdots \ \breve{\mathbf{H}}_M]$, $\breve{\mathbf{H}}_m = \mathrm{Toeplitz}(\breve{\mathbf{h}}_m^T, N, 1)$. Next, we need to specify the second term in (23). Assuming that the variable components $\breve{\mathbf{h}}_m$ are spatially uncorrelated with each other, we have

$$E[\breve{\mathbf{H}}^H \mathbf{Q}^{(k)} \breve{\mathbf{H}}] = \mathrm{diag}(E[\breve{\mathbf{H}}_1^H \mathbf{Q}^{(k)} \breve{\mathbf{H}}_1] \ \cdots \ E[\breve{\mathbf{H}}_M^H \mathbf{Q}^{(k)} \breve{\mathbf{H}}_M]). \tag{24}$$

From the property of the Kronecker product†, a submatrix is expressed as $E[\breve{\mathbf{H}}_m^H \mathbf{Q}^{(k)} \breve{\mathbf{H}}_m] = \mathrm{mat}\left(E[\breve{\mathbf{H}}_m^T \otimes \breve{\mathbf{H}}_m^H] \mathrm{vec}(\mathbf{Q}^{(k)})\right)$. Since the correlation matrix $\mathbf{\Omega}_{\breve{h}}^T = E[\breve{\mathbf{H}}_m^T \otimes \breve{\mathbf{H}}_m^H]$ can be assumed to be the same for all m, we have

$$E[\breve{\mathbf{H}}^H \mathbf{Q}^{(k)} \breve{\mathbf{H}}] = \mathbf{I}_M \otimes \mathrm{mat}\left(\mathbf{\Omega}_{\breve{h}}^T \mathrm{vec}(\mathbf{Q}^{(k)})\right). \tag{25}$$

After some arithmetic, we have

$$\mathbf{\Omega}_{\breve{h}} = \mathrm{Toeplitz}(\mathbf{\Omega}_{\breve{h}b}, N, N+L-1) \in \mathbb{R}^{N^2 \times (N+L-1)^2},$$

† $\mathrm{vec}(\mathbf{AXB}) = (\mathbf{B}^\mathrm{T} \otimes \mathbf{A}) \mathrm{vec}(\mathbf{X})$ is used.

$$\boldsymbol{\Omega}_{\check{h}b} = [\ \underbrace{\mathbf{I}_N, \mathbf{0}_{N\times(L-1)}}_{N+L-1 \text{ columns}},\ \underbrace{\mathbf{0}_{N\times 1}, \mathbf{I}_N, \mathbf{0}_{N\times(L-2)}}_{N+L-1 \text{ columns}},\ \cdots,\ \underbrace{\mathbf{0}_{N\times(L-1)}, \mathbf{I}_N}_{N+L-1 \text{ columns}}\ ] \in \mathbb{R}^{N\times L(N+L-1)}. \quad (26)$$

Consequently, using (23), (25), and (26), we obtain $\boldsymbol{\Omega}_{sos,k}$.

The second matrix $\boldsymbol{\Omega}_{\mathrm{p}} = E[\mathbf{u}_{\mathrm{p}}\mathbf{u}_{\mathrm{p}}^H]$ depends on the transmission signal of PT. In this paper, we consider two types of the signal: OFDM signal and narrow-band signal.
(1) *OFDM signal.* We assume that the parameters of the primary OFDM signal $u_{\mathrm{p}}[n]$ are the same as those of the secondary OFDM signal $u_j[n]$. Furthermore, for simplicity, the transmission timing of the primary OFDM symbol is assumed to be identical to that of the secondary one. Due to the similarity of $u_{\mathrm{p}}[n]$ and $u_j[n]$, it is obvious that

$$\boldsymbol{\Omega}_{\mathrm{p}} = P_p/P_s \boldsymbol{\Omega}_j. \quad (27)$$

where P_p is the power of the transmitted signal $u_{\mathrm{p}}[n]$ of PT.
(2) *Narrow-band signal.* A narrow-band signal is modeled by a second-order auto-regressive (AR) process [12]. Then, the transmitted signal $u_{\mathrm{p}}[n]$ can be represented as

$$u_{\mathrm{p}}[n] = -a_1 u_{\mathrm{p}}[n-1] - a_2 u_{\mathrm{p}}[n-2] + w[n] \quad (28)$$

where a_i denotes an AR parameter and $w[n]$ is a white Gaussian process $\mathcal{CN}(0, P_p)$. Then, the correlation matrix $\boldsymbol{\Omega}_{\mathrm{p}}$ is given by:

$$\boldsymbol{\Omega}_{\mathrm{p}} = \begin{bmatrix} r[0] & r[1] & \cdots & r[N_{\mathrm{p}}-1] \\ r[1] & r[0] & \cdots & r[N_{\mathrm{p}}-2] \\ \vdots & & \ddots & \vdots \\ r[N_{\mathrm{p}}-1] & r[N_{\mathrm{p}}-2] & \cdots & r[0] \end{bmatrix} \quad (29)$$

where $N_{\mathrm{p}} = N + L_p$ and $r[m]$ denotes the autocorrelation function of the AR process, which is given by [13]

$$r[m] = \frac{P_p\left\{(p_2^2-1)p_1^{m+1} - (p_1^2-1)p_2^{m+1}\right\}}{(p_2-p_1)(p_1p_2+1)} \quad (30)$$

where $p_1, p_2 = \frac{1}{2}\left(-a_1 \pm \sqrt{a_1^2 - 4a_2}\right)$.

3.1.3 Relay Transmission Power

From (5), the total transmit power P_r at the relays can be derived as†

$$\begin{aligned} P_r &= E\left[\sum_{m=1}^{M} \mathrm{tr}(\mathbf{y}_m\mathbf{y}_m^H)\right] \\ &= \sum_{m=1}^{M}\sum_{i=1}^{2} \mathrm{tr}\left\{\mathbf{R}_m\left(\tilde{\mathbf{F}}_{m,i}\boldsymbol{\Omega}_i\tilde{\mathbf{F}}_{m,i}^H + \boldsymbol{\Omega}_{sos,m,Y} + \sigma_r^2\mathbf{I}\right)\mathbf{R}_m^H\right\} \\ &= \sum_{m=1}^{M} \mathrm{vec}(\mathbf{R}_m^T)^T\mathbf{Y}_m\mathrm{vec}(\mathbf{R}_m^H) \\ &= \mathbf{r}^H\boldsymbol{\Phi}_Y\mathbf{r} \end{aligned} \quad (31)$$

†We use the fact that $\mathbf{F}_{m,i}\tilde{\mathbf{R}}_m = \mathbf{R}_m\tilde{\mathbf{F}}_{m,i}$ and $\mathbf{H}_m\tilde{\mathbf{R}}_m = \mathbf{R}_m\tilde{\mathbf{H}}_m$.

where $\boldsymbol{\Omega}_{sos,m,Y} = E[\tilde{\mathbf{H}}_m\boldsymbol{\Omega}_{\mathrm{p}}\tilde{\mathbf{H}}_m^H]$ and

$$\begin{aligned} \boldsymbol{\Phi}_Y &= \mathrm{diag}([\mathbf{E}_2\mathbf{Y}_1^*\mathbf{E}_2^H\ \cdots\ \mathbf{E}_2\mathbf{Y}_M^*\mathbf{E}_2^H]), \\ \mathbf{E}_2 &= [\ \underbrace{\mathbf{I}_{L_r}, \mathbf{0}_{L_r\times(N+L-2)}}_{N+L+L_r-2\text{columns}},\ \underbrace{\mathbf{0}_{L_r\times 1}, \mathbf{I}_{L_r}, \mathbf{0}_{L_r\times(N+L-3)}}_{N+L+L_r-2\text{columns}}, \\ &\quad \cdots,\ \underbrace{\mathbf{0}_{L_r\times(N+L-2)}, \mathbf{I}_{L_r}}_{N+L+L_r-2\text{columns}}\ ] \in \mathbf{G}^{L_r\times(N+L-1)(N+L+L_r-2)}, \\ \mathbf{Y}_m &= \mathbf{I}_{N+L-1} \otimes \left(\sum_{i=1}^{2}\tilde{\mathbf{F}}_{m,i}\boldsymbol{\Omega}_i\tilde{\mathbf{F}}_{m,i}^H + \boldsymbol{\Omega}_{sos,m,Y} + \sigma_r^2\mathbf{I}\right), \\ \tilde{\mathbf{F}}_{m,i} &= \mathrm{Toeplitz}(\mathbf{f}_{m,i}^T, N+L_r+L-2, 1), \\ \tilde{\mathbf{H}}_m &= \mathrm{Toeplitz}(\mathbf{h}_m^T, N+L_r+L-2, 1). \end{aligned} \quad (32)$$

The SOS matrix $\boldsymbol{\Omega}_{sos,m,Y}$ can be specified in the same manner as $\boldsymbol{\Omega}_{sos,k}$ in (23): the differences are that the second parameter of the Toeplitz matrices $\bar{\mathbf{H}}$ and $\check{\mathbf{H}}$ is $N + L + L_r - 2$ instead of N; and $\mathbf{Q}^{(k)}$ is replaced by $\boldsymbol{\Omega}_{\mathrm{p}}$ in (27) or (29).

3.2 *Method 1:* Worst Subcarrier SINR Maximization subject to Relay Transmission Power Constraint††

Now, we propose the first FF-BF design method for QoS enhancement of the secondary network. The bit errors often occur at the subcarriers which have low SINR in ordinary OFDM systems. Thus, it is reasonable to consider the maximization of the worst subcarrier SINR of the secondary transceivers in OFDM-based CR networks. At the same time, the total relay transmission power P_r should be limited to a certain level to reduce the interference from relays to PR. Mathematically, the optimization problem can be formulated as:

$$\max_{\mathbf{r}}\ \min_{i,k}\ SINR_{i,k} \quad \text{s. t.} \quad P_r \le P_{r,max} \quad (33)$$

where $P_{r,max}$ is the maximum total relay transmission power and $SINR_{i,k}$ is the kth subcarrier SINR of SU_i defined by

$$SINR_{i,k} = \frac{P_{S,k}}{P_{I,i,k} + P_{N,i,k}}. \quad (34)$$

From (14), (18), (20) and (31), the optimization problem can be reformulated as follows:

$$\begin{aligned} &\max_{\mathbf{r}}\ \min_{i,k}\ \frac{\mathbf{r}^H\boldsymbol{\Phi}_{S,k}\mathbf{r}}{\mathbf{r}^H\boldsymbol{\Phi}_{IN,i,k}\mathbf{r} + \sigma_i^2} \\ &\text{s. t.} \quad \mathbf{r}^H\boldsymbol{\Phi}_Y\mathbf{r} \le P_{r,max} \end{aligned} \quad (35)$$

where $\boldsymbol{\Phi}_{IN,i,k} = \boldsymbol{\Phi}_{I,i,k} + \boldsymbol{\Phi}_{N,i,k}$. To solve this problem, we take the approach in [7]. By introducing a semidefinite matrix $\mathcal{R} = \mathbf{r}\mathbf{r}^H$ and a slack variable t, the above problem can be rewritten as follows:

$$\begin{aligned} &\max \quad t \\ &\text{s. t.} \quad \mathrm{tr}(\boldsymbol{\Phi}_Y\mathcal{R}) \le P_{r,max} \\ &\qquad \mathrm{tr}((\boldsymbol{\Phi}_{S,k} - t\boldsymbol{\Phi}_{IN,i,k})\mathcal{R}) \ge \sigma_i^2 t, \quad k = 1, \cdots, N \\ &\qquad \mathcal{R} \succeq 0 \qquad\qquad\qquad\qquad\qquad i = 1, 2 \end{aligned} \quad (36)$$

††Method 1 proposed here has already been presented in our conference paper [14], but only in the case of the full CSI($\alpha = 0$) and the OFDM-based PU signal.

where the rank one constraint of $\mathcal{R}$ is dropped by semidefinite relaxation [15]. To obtain the solution of the above quasi-convex problem, we consider its corresponding feasibility problem [7], [3]:

$$\begin{aligned} &\text{find} \quad \mathcal{R} \\ &\text{s. t.} \quad \mathrm{tr}(\mathbf{\Phi}_Y \mathcal{R}) \le P_{r,max} \\ &\qquad \mathrm{tr}((\mathbf{\Phi}_{S,k} - t\mathbf{\Phi}_{IN,i,k})\mathcal{R}) \ge \sigma_i^2 t, \quad k = 1, \cdots, N \\ &\qquad \mathcal{R} \succeq 0. \qquad\qquad\qquad\qquad i = 1, 2 \end{aligned} \tag{37}$$

Note that for a given t, the above feasibility problem is a semidefinite problem (SDP). Such SDPs can be efficiently solved by free software packages, e.g., the SDP solver SeDuMi whose computational complexity is at most $O((N + (Lr \times M)^2)^{3.5})$ [16] in conjunction with the parser YALMIP [17]. To obtain a solution, we employ the following bisection algorithm:

Step1: Set an appropriate interval (t_L, t_R) and an error tolerance ϵ for t.
Step2: Set $t = (t_L + t_R)/2$.
Step3: Solve the SDP (37) for t. If it is feasible, set $t_L = t$. Otherwise, set $t_R = t$.
Step4: Repeat Steps 1 and 2 until $(t_R - t_L) < \epsilon$.

The resulting solution $\mathcal{R}$ may not be a rank-1 matrix in general. To obtain a good approximation solution $\mathbf{r}$ from $\mathcal{R}$, we adopt a randomization technique [3]: first, generate candidates by $\mathbf{r}_n = \sqrt{\frac{P_{r,max}}{\mathbf{v}_n^H \mathbf{\Phi}_Y \mathbf{v}_n}} \mathbf{U}\mathbf{\Lambda}\mathbf{v}_n$ for $n = 1, 2, \cdots, N_r$ where eigendecomposition $\mathcal{R} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H$ and random vectors $\mathbf{v}_n \sim \mathcal{CN}(0, \mathbf{I}_{M \times L_r})$; then choose the best one which maximizes the minimum SINR.

3.3 *Method 2:* Relay Transmission Power Minimization subject to Per-Subcarrier SINR Constraints

Next, we propose the second FF-BF design method which suppresses the interference from SU_i to PR. Specifically, the relay transmission power P_r is minimized while SINR per OFDM subcarrier is limited. This optimization problem can be formulated as:

$$\min_{\mathbf{r}} \quad P_r \quad \text{s. t.} \quad SINR_{i,k} \ge \gamma, \quad k = 1, \cdots, N \quad i = 1, 2 \tag{38}$$

where γ is the required minimum each subcarrier SINR.

From (14), (18), (20) and (31), the above optimization problem can be restated as

$$\begin{aligned} &\min_{\mathbf{r}} \qquad \mathbf{r}^H \mathbf{\Phi}_Y \mathbf{r} \\ &\text{s. t.} \quad \frac{\mathbf{r}^H \mathbf{\Phi}_{S,k} \mathbf{r}}{\mathbf{r}^H \mathbf{\Phi}_{IN,i,k} \mathbf{r} + \sigma_i^2} \ge \gamma, \quad k = 1, \cdots, N \quad i = 1, 2. \end{aligned} \tag{39}$$

Again, as in earlier, we introduce a semidefinite matrix $\mathcal{R}$ and drop its rank-one constraint, then we can rewrite the problem as

$$\begin{aligned} &\min_{\mathcal{R}} \quad \mathrm{tr}(\mathbf{\Phi}_Y \mathcal{R}) \\ &\text{s. t.} \quad \mathrm{tr}((\mathbf{\Phi}_{S,k} - \gamma\mathbf{\Phi}_{IN,i,k})\mathcal{R}) \ge \sigma_i^2 \gamma, \quad k = 1, \cdots, N \\ &\qquad \mathcal{R} \succeq 0. \qquad\qquad\qquad\qquad i = 1, 2 \end{aligned} \tag{40}$$

For a given γ, (40) is an SDP problem. To solve it, we can use YALMIP, SeDuMi and the randomization technique.

4. Numerical Results

4.1 Simulation Conditions

To evaluate the proposed methods, we present simulation results. Unless otherwise stated, the following parameters are used. The channel impulse response coefficients are modeled as zero-mean complex Gaussian variables with unit variance. The channel mean $\bar{\mathbf{h}}_m$ is generated according to $\mathcal{CN}(\mathbf{0}, \mathbf{I})$ only once per simulation trial. All results are obtained by averaging over 500 trials. We assumed that the received timings at an SU from all relays are synchronized unlike asynchronous relay networks in [18], and the maximum relay filter length is $L_r = 3$. The AR parameters are $a_1 = -0.1$ and $a_2 = -0.01$. The number of vectors for randomization is $N_r = 1000$. The noise power at the relays and receivers are $\sigma_r^2 = \sigma_i^2 = 1$, and the transmission powers are P_s=20 dB and P_p=23 dB higher than the noise power. The maximum relay transmission power is $P_R = 15$ dB higher than the noise power and $P_{r,max} = P_R(N + L - 1)$. The uncertainty parameter is $\alpha = 0$. The performance of PU is evaluated by BER when SUs operate in the second phase.

4.2 Worst Subcarrier SINR Maximization

First, we show the results of the SINR maximization FF-BF design where the number of OFDM subcarriers is $N = 32$ and the channel length is $L = 3$. Figure 2 shows that the worst subcarrier SINR of the overall secondary network versus the length of the relay filters for various α when the number of relays is M=5 or 10. Note that $\alpha = 0$ corresponds to the case where the full CSI is available, and a larger α implies a weaker correlation among channels. In all cases, SINR improves with the relay filter length. Especially, the SINR of the case with partial CSI ($\alpha = 0.1$) approaches that with full CSI ($\alpha = 0$) when $M = 10$.

Figure 2 also shows the results when the primary trans-

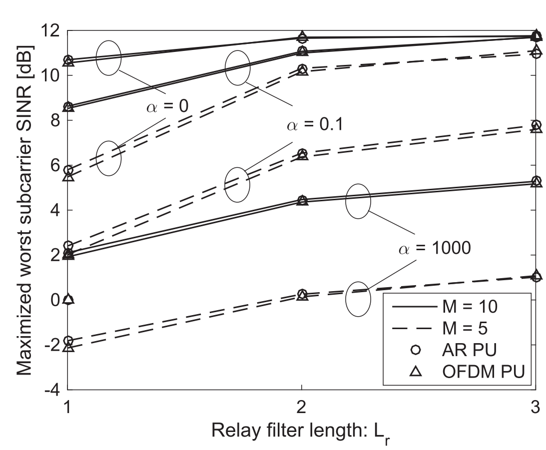


Fig. 2 Maximized worst subcarrier SINR.

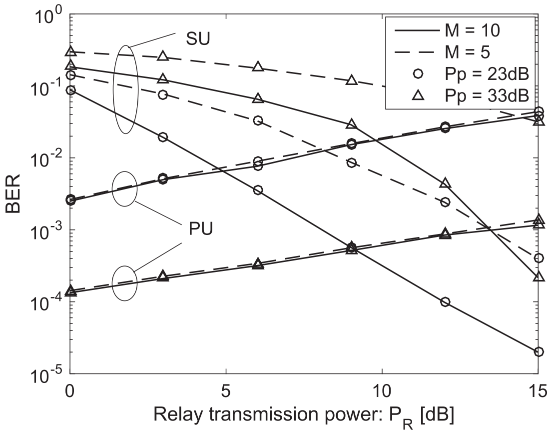
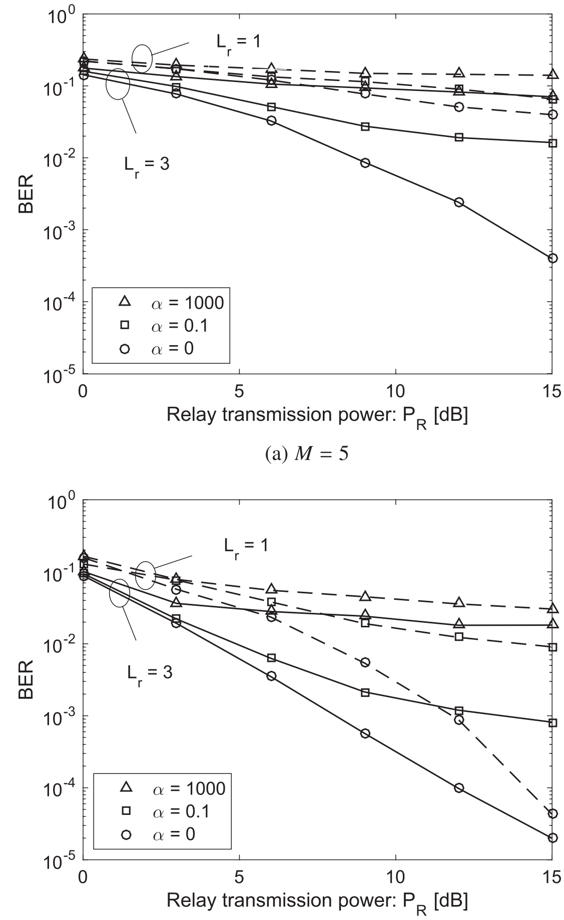


Fig. 3 Average BER of PU and SU versus relay transmission power.



(a) $M = 5$

(b) $M = 10$

Fig. 4 Average BER comparison between FF-BF($L_r = 3$) and AF-BF($L_r = 1$) for $\alpha = 0, 0.1, 1000$.

mission signal $u_{\mathrm{p}}[n]$ is either OFDM signal or AR process. Although there is a slight difference between two cases of PU, the same performance trend can be observed in both cases. For all results shown in the rest of this section, we have confirmed that almost the same performance is obtained regardless of the type of the primary transmission signal. For the simplicity, we present only the results of the OFDM signal in the following figures except Fig. 4.

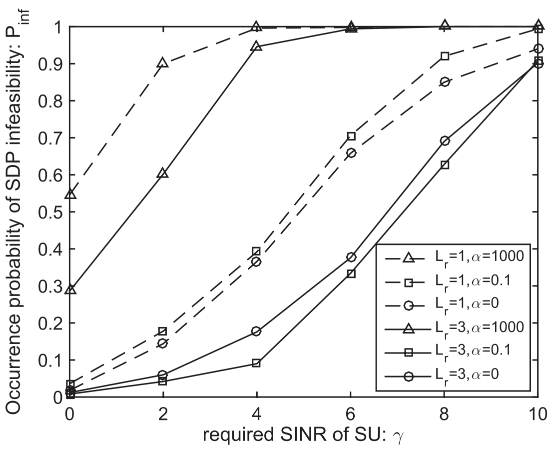
The influence of the relay transmission power P_R on the BER performance of PU and SU is shown in Fig. 3 where *PT* source power is P_p=23 dB or 33 dB and the number of relays $M = 5$ or 10. PU employs the identical OFDM system with SUs. As can be seen from the figure, the BER of SU becomes better as the relay power P_R increases even if the *PT* source power P_p increases. This suggests that the relays can suppress the interference from *PT* significantly. It can also be seen that the BER of PU is less sensitive to P_R. This is because relays always transmit with low power due to the limitation of interference to *PR*, e.g., the maximum relay transmission power $P_R = 15$ dB is much less than the *PT* source power $P_p = 23$ dB. Also, the BER of SU improves with M, while that of PU is independent of M because it is dominated by P_p.

In Fig. 4, SU's BER performance of the FF-BF with $L_r = 3$ is compared with that with $L_r = 1$ (corresponding to AF-BF) for a various number of relays M and uncertainty parameter α, where the PU signal is an AR process. We observe significant performance improvement of FF-BF with $L_r = 3$ over AF-BF with $L_r = 1$, for example, when $M = 5$ and $\alpha = 0$, FF-BF achieves a BER of below 10^{-3} while AF-BF can only achieve a BER of above 10^{-2}. Moreover, the proposed method is efficient even if only partial CSI is available, for example, when $M = 10$, $\alpha = 0.1$, and $P_R = 15$ dB, FF-BF achieves a BER of 10^{-3} while AF-BF achieves a BER of 10^{-2}. It can also be observed that the performance of the proposed method is enhanced when the number of relays M is large. This is because more relays are used, larger BF gain is obtained.

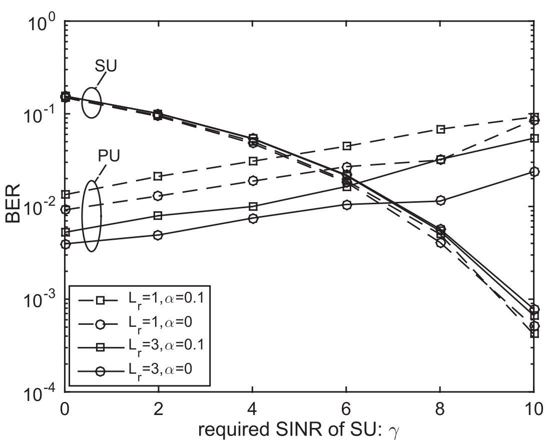
4.3 Relay Transmission Power Minimization

Next, we evaluate the second FF-BF design which minimizes the relay transmission power. To save simulation time, we considered a smaller system with $N = 16$ and $L = 2$. Note that the required SINR cannot always be achieved for all subcarriers. In other words, SDP can be infeasible, in which case we cannot obtain any solution by SDP. In Figs. 5(a) and 6(a), we show the occurrence probability of the SDP infeasibility P_{inf} versus the required minimum subcarrier SINR γ for $M = 5$ and 10. As can be seen, the SDP infeasibility can be reduced efficiently by using filters with $L_r = 3$ and increasing the number of relays M. Whereas it is barely feasible when we have limited CSI ($\alpha = 1000$).

In Figs. 5(b) and 6(b), the BER performances for $M = 5$ and 10 are shown where the result of $\alpha = 1000$ is not shown because SDP becomes infeasible for high γ. We observe that the BER performance of SU is independent of L_r, α, and M, and it is dominated by the minimum subcarrier SINR of SU γ. Also, the BER of PU improves with M. It can also be seen that the improvement of the PU's BER per-

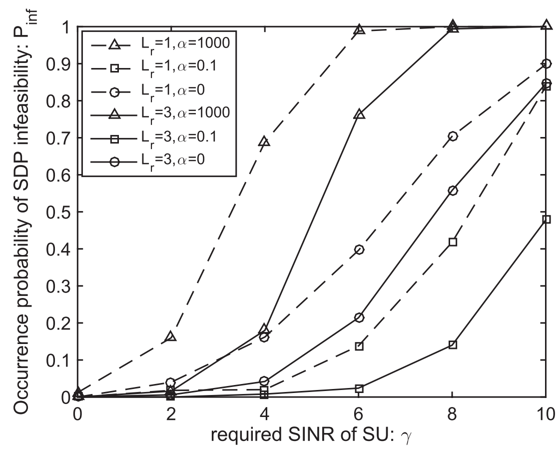


(a) Occurrence probability of SDP infeasibility

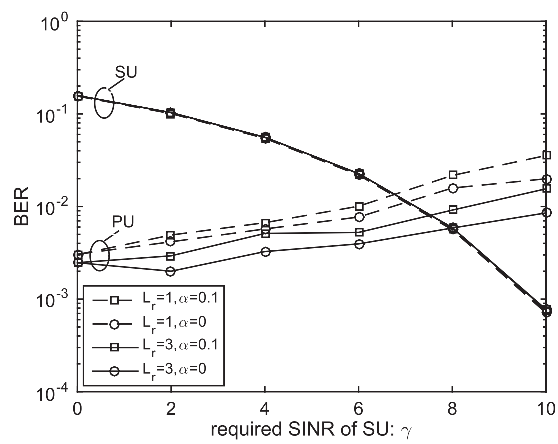


(b) BER performance

Fig. 5 Probability of the SDP infeasibility and minimized relay transmission power versus the required min subcarrier SINR of SUs when $M = 5$.



(a) Occurrence probability of SDP infeasibility



(b) BER performance

Fig. 6 Probability of the SDP infeasibility and minimized relay transmission power versus the required min subcarrier SINR of SUs when $M = 10$.

formance of FF-BF over AF-BF can be obtained, for example, when $M = 10$ and $\alpha = 0$, FF-BF gains 3 dB reduction compared to AF-BF to achieve a BER of 10^{-2}.

5. Conclusion

In this paper, we have proposed two FF-BF relay design methods for OFDM-based cognitive networks. The first method optimizes FF-BF filters at the relays by maximizing the minimum worst subcarrier SINR of secondary transceivers subject to the total relay transmission power constraint. The second method optimizes the filters by minimizing the relay transmission power subject to the subcarrier SINR constraints. Simulation results showed that the proposed FF-BF outperforms the AF-BF relay and improves overall system performance because of its SINR improvement and interference suppression capability. The scenario considered in this paper is restrictive since the secondary network operates in half duplex mode. Therefore, it is worthwhile to study full duplex systems as they can improve the bandwidth efficiency. In this case, the suppression of inter-relay interference and self-interference by directional antennas or additional interference cancellers might be important [19], [20]. Also, challenging topics for future research include the performance improvement by FF relay design based on other criteria such as MMSE, and FF relay design for single-carrier with frequency domain equalization (SC-FDE) systems.

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