# Time-Domain Equalizer Using Neural Network Without Known Training Signals for OFDM Systems Without CP

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## 1. Introduction

Recently, machine learning has been applied to various problems in wireless physical layer [1]. In [2], an extreme learning machine (ELM)-based signal detector for OFDM systems has been proposed. The ELM-based detector is designed by online training using the received samples obtained through only a current channel. It is reported that the detector is superior to the deep neural network (DNN)-detector in [3] designed by pre-training using samples generated by a channel model. However, the ELM-detector requires to transmit enormous known signals for online training. In this article, we propose a novel time-domain equalizer (TEQ) using an online-trained neural network (NN) for OFDM systems without cyclic prefix (CP). Unlike [2], the proposed TEQ requires no transmission of known signals for online training.

## 2. Proposed Time-Domain Equalizer

We consider an OFDM system with $N$ subcarriers. Data symbols $s_k$ are grouped into a data block $\mathbf{s}_n \in \mathbb{C}^N$. At the transmitter with a single antenna, a time-domain signal of the $n$th block obtained by $N$-point inverse discrete Fourier transform (IDFT) of $\mathbf{s}_n$ is transmitted, $\mathbf{x}_n = \mathbf{F}^H \mathbf{s}_n = \left[x_{nN}\ x_{nN-1}\ \cdots\ x_{nN-(N-1)}\right]^T \in \mathbb{C}^N$ where $\mathbf{F}$ is the DFT matrix. We assume that the desired signal is $x_{k-d}$, where $d$ is an arbitrary decision delay.

At the receiver with $N_r$ antennas, the proposed TEQ uses $L_w$ successive received samples $\mathbf{r}_k = \mathbf{h}_d x_{k-d} + \check{\mathbf{H}}\check{\mathbf{x}}_k + \mathbf{n}_k \in \mathbb{C}^{N_r L_w}$, where $\mathbf{h}_d$ is the $(d+1)$th column vector of the current channel matrix $\mathbf{H} \in \mathbb{C}^{N_r L_w \times (L_w + L_h)}$, $\check{\mathbf{H}}$ is a matrix removed $\mathbf{h}_d$ from $\mathbf{H}$, $\check{\mathbf{x}}_k$ is a transmitted signal vector, $\mathbf{n}_k$ is a Gaussian noise vector. The purpose of the proposed TEQ is to detect $x_{k-d}$ from $\mathbf{r}_k$ by suppressing interference and noise components. This article assumes that the receiver obtains $\mathbf{H}$ by blind channel estimation.

In the proposed TEQ, the input signal $\mathbf{r}_k$ is processed in two parts. First, the maximum ratio combining is performed as $u_k = \mathbf{h}_d^H \mathbf{r}_k = \|\mathbf{h}_d\|^2 x_{k-d} + \mathbf{h}_d^H \check{\mathbf{H}}\check{\mathbf{x}}_k + \mathbf{h}_d^H \mathbf{n}_k$. Second, the desired signal is removed as $\check{\mathbf{r}}_k = \mathbf{H}_d^{\perp} \mathbf{r}_k = \mathbf{H}_d^{\perp} \check{\mathbf{H}}\check{\mathbf{x}}_k + \mathbf{H}_d^{\perp} \mathbf{n}_k$, where $\mathbf{H}_d^{\perp} = \mathbf{I}_{N_r L_w} - \mathbf{h}_d \mathbf{h}_d^H / \|\mathbf{h}_d\|^2$ is orthogonal to $\mathbf{h}_d$, and $\mathbf{I}$ is the identity matrix. Then the processed signal $\check{\mathbf{r}}_k$ is fed into a NN $F(\cdot)$ with adjustable parameters $\mathbf{w}$. The output of NN is $z_k = F(\check{\mathbf{r}}_k; \mathbf{w})$. The TEQ output is given by $y_k = u_k - z_k$, and $\tilde{y}_k = y_k / \|\mathbf{h}_d\|^2$ are grouped into a block $\tilde{\mathbf{y}}_n$. Finally, DFT is applied, and the estimated block $\hat{\mathbf{s}}_n$ can be obtained by the hard-decision of $\tilde{\mathbf{Y}}_n = \mathbf{F}\tilde{\mathbf{y}}_n$.

Note that the desired signal in $y_k$ is never affected by NN because $z_k$ contains only the interference and noise components. Thus, the NN is trained by minimizing $J(\mathbf{w}) = |y_k|^2$, and the TEQ can suppress interference and noise while the desired signal component is kept unchanged. In the proposed TEQ, no known training signals are required to train the NN.

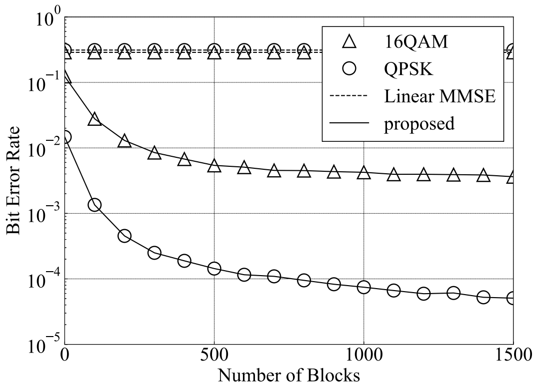


**Fig. 1** Effect of the number of training blocks on BER.

## 3. Simulation Result

In the simulation, the order of the CIR was $L_h = 8$, $N = 64$, $N_r = 2$, $L_w = 45$, and $d = \lfloor (L_w + L_h)/2 \rfloor$. We used a NN with three full-connected layers, and the activation function is ReLU. The optimizer was the Adam optimizer with the learning rate of $5 \times 10^{-5}$. Figure 1 shows the BER performance as a function of the number of training blocks, where SNR is 40 dB. Unlike a constrained linear MMSE-TEQ, the BER of the proposed TEQ decreases without known training signals as the number of training blocks increases. The computational complexities per block required by the proposed TEQ, [3], and [2] are about $1 \times 10^7$, $3 \times 10^6$, and $3 \times 10^5$ real multiplications, respectively.

## 4. Conclusion

We proposed a novel TEQ using NN without known signals for online training for OFDM systems without CP.

**References**

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