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Paper

Distributed minimum variance equalization in wireless sensor networks

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Received January 4, 2021; Revised March 19, 2021; Published July 1, 2021

Abstract: This paper proposes a novel distributed blind adaptive equalization algorithm for sensor networks, in which each node estimates the data disturbed by inter-symbol interference using time-domain filtering. We apply the minimum variance distortionless response, which is known to be useful for centralized blind equalization, to distributed equalization with multiple distortionless response constraints. Unlike conventional methods, in the proposed approach, each node requires only one filter and sends one signal to the other nodes. Simulation results show the superior performance of the proposed algorithm compared with the conventional algorithm.

Key Words: distributed equalization, blind equalization, minimum variance distortionless response, sensor network, channel estimation, average consensus

1. Introduction

Distributed signal processing in wireless sensor networks, where distributed communication nodes jointly perform data processing, has been extensively studied in recent years [1, 2]. In distributed signal processing, even if some nodes become defective, the network can still be reconfigured by the remaining nodes. A significant advantage of this method is that it does not require a fusion center, which is critical for centralized signal processing; therefore, devices can be simpler.

Distributed equalization/estimation has been extensively studied, and it is worth investigating further [3–7]. Its potential applications are widespread in many areas, such as environmental monitoring, factory automation, and drone communications. In distributed equalization [7], each node that is distributed over a wide area ranging from a few meters to several hundred meters estimates the data symbols that are broadcast by the source. The data symbols suffer from inter-symbol interference (ISI), which is caused by signal propagation through a frequency-selective channel from the source to each node. Each node performs time-domain equalization using an adaptive finite-impulse response (FIR) filter to suppress ISI, and communicates with its neighboring nodes to accomplish equalization in a distributed manner. A disadvantage of most existing methods is that they require the transmission of training (reference) signals that waste valuable bandwidth resources.

Distributed *blind* adaptive equalization has received significant attention because it avoids the transmission of bandwidth-consuming training signals. Thus far, various adaptive algorithms have been proposed in the literature [7–11]. Abdolee et al. [7] proposed a distributed equalization algorithm based

Nonlinear Theory and Its Applications, IEICE, vol. 12, no. 3, pp. 442–452 ©IEICE 2021 DOI: 10.1587/nolta.12.442

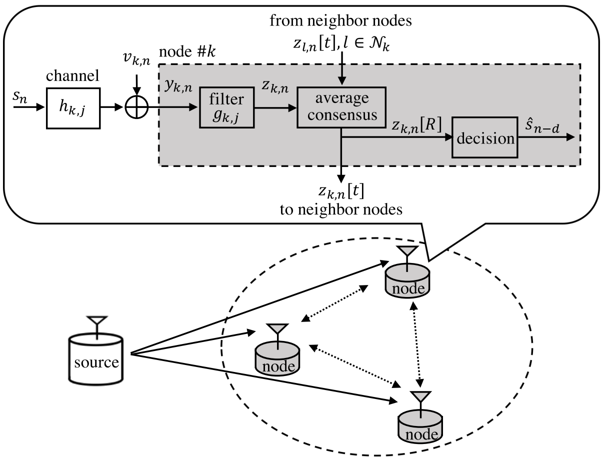

Fig. 1. System model of distributed equalization.

on the constant modulus criterion and demonstrated its superiority over the corresponding centralized equalization algorithm. Bertrand et al. [8] proposed a linearly constrained minimum-variance-based distributed beamforming method that actively exploits the spatial diversity introduced by the distributed nodes. Yu et al. [9] proposed an indirect distributed equalization method where the channels are estimated first; then, the equalizers are determined recursively. A common disadvantage of the above approaches is that the nodes need to exchange many continuous-valued signals, such as their received signals, their filter coefficients, and estimated channels. This exchange requires significant computation and communication resources. Liu et al. [10] proposed the distributed generalized Sato algorithm (d-GSA), which requires sending only one signal per node. It was reported that the performance of the d-GSA degrades with the increasing number of differences in the channels between the source and other nodes. This limits its applicability because each channel varies depending on whether the nodes are randomly distributed over a wide area. Sugitani et al. [11] proposed a distributed mutually referenced equalization (d-MRE) algorithm, in which all the channels are assumed to be different from each other. It was reported that the performance of d-MRE is much better than that of d-GSA. However, the d-MRE has two disadvantages: 1) each node requires two FIR filters and sends two signals to other nodes, and 2) the bit error performance is not satisfactory, even in the presence of many nodes.

The minimum variance distortionless response (MVDR) principle is known to achieve excellent bit error performance for centralized blind equalization [13]. As MVDR imposes a single distortionless response constraint for all nodes, it requires a fusion center that collects all the received signals of the nodes. Hence, it cannot be applied to distributed equalization as is. To the best of our knowledge, the application of MVDR to distributed equalization has not been previously reported.

In this study, we apply the MVDR principle to distributed equalization by decomposing a single constraint into multiple constraints and deriving an adaptive equalization algorithm. The proposed algorithm is simpler than the d-MRE because each node requires only one FIR filter and sends one signal to the other nodes. Simulation results reveal that the bit error performance of the proposed algorithm is superior to that of the d-MRE.

2. Problem formulation

2.1 Communication model

Let us consider a networked system with K distributed nodes and one single source (see Fig. 1). A data symbol s_n, which is sent from the source at time n, passes through the frequency-selective channels, thereby causing ISI. Then, the received signal at time n by the kth node is given by

$$y_{k,n} = \sum_{j=0}^{L} h_{k,j} s_{n-j} + v_{k,n} = h_{k,d} s_{n-d} + \sum_{\substack{j=0 \\ j \neq d}}^{L} h_{k,j} s_{n-j} + v_{k,n}, \quad k = 1, 2 \ldots K, \tag{1}$$

where $h_{k,j}$ is the impulse response of the channel from the source to the kth node, L is the order of the channel impulse responses, $v_{k,n}$ is the additive white Gaussian noise, and d is the decision delay that affects the system performance. On the rightmost side of (1), the first and second terms represent the desired component and the ISI component, respectively.

Each node has one adaptive FIR filter with length M, and processes its received signal $y_{k,n}$ by the filter as follows:

$$z_{k,n} = \sum_{j=0}^{M-1} g_{k,j}^* y_{k,n-j} = \mathbf{g}_k^H \mathbf{y}_{k,n}, \tag{2}$$

where $g_{k,j}$ is the filter coefficient of the FIR filter, $\mathbf{g}_k = [g_{k,0} \cdots g_{k,M-1}]^T \in C^{M\times 1}$, and $\mathbf{y}_{k,n} = [y_{k,n} \cdots y_{k,n-M+1}]^T \in C^{M\times 1}$. Here, the superscripts T and H denote the transpose and the conjugate transpose of a vector (matrix), respectively. For convenience, we represent $\mathbf{y}_{k,n}$ in a matrix form as

$$\mathbf{y}_{k,n} = \mathbf{H}_k \mathbf{s}_n + \mathbf{v}_{k,n}, \tag{3}$$

where $\mathbf{H}_k = \begin{bmatrix} h_{k,0} \cdots h_{k,L} \cdots & 0 \\ \vdots \ \ddots \ \ddots \ \ddots & \vdots \\ 0 \ \cdots h_{k,0} \cdots & h_{k,L} \end{bmatrix}$ is an $M \times (L+M)$ Toeplitz matrix, $\mathbf{s}_n = [s_n \ldots s_{n-M-L+1}]^T \in C^{(L+M)\times 1}$, and $\mathbf{v}_{k,n} = [v_{k,n} \ldots v_{k,n-M+1}]^T \in C^{M\times 1}$. The purpose of the FIR filter is to suppress the effect of ISI for the proper estimation of the data symbol s_{n-d}. To suppress ISI, the filter coefficients $\mathbf{g}_k$ should be updated appropriately using the adaptive algorithm described in Sect. 3.2.

Each node obtains the averaged filter output defined by $z_n = \frac{1}{K}\sum_{k=1}^{K} z_{k,n}$ by the average consensus technique described in Sect. 2.2. By making hard decision on z_n, we obtain the estimation $\hat{s}_{n-d}$ of the data symbol s_{n-d}. As described later, the average z_n is necessary for updating the filter coefficients.

2.2 Average consensus

We assume that each node is not always wirelessly connected with the other nodes located far from it. Thus, in some network topologies, the nodes cannot obtain z_n directly. The deployment of a fusion center to obtain z_n is undesirable in terms of complexity increase. To overcome this issue, we apply an average consensus technique [12] to estimate z_n at each node in an iterative manner without a fusion center. Let $z_{k,n}[t]$ be the estimate of z_n in the kth node at the tth iteration. The kth node sends one signal $z_{k,n}[t]$ to the neighbor nodes through wireless links, and receives $z_{i,n}[t]$ of the neighbor nodes $i \in \mathcal{N}_k$, where $\mathcal{N}_k$ is the set of nodes wirelessly connected to the kth node. In the following, we assume that the communication channels between nodes are noiseless and distortionless. We set the initial value as $z_{k,n}[0] = z_{k,n}$. Then, we update $z_{k,n}[t]$ iteratively as follows:

$$z_{k,n}[t+1] = \sum_{i\in\mathcal{N}_k} w_{k,i} z_{i,n}[t], \tag{4}$$

where $w_{k,i}$ is the link weight between the ith and kth nodes. Let us define the weight matrix as $\mathbf{W} = \begin{bmatrix} w_{1,1} & \cdots & w_{1,K} \\ \vdots & w_{k,k} & \vdots \\ w_{K,1} & \cdots & w_{K,K} \end{bmatrix} \in Q^{K\times K}$. If $\mathbf{W}$ satisfies the following conditions:

$$w_{i,j} = 0 \text{ if } j \notin \mathcal{N}_i, \quad \mathbf{W1} = \mathbf{1}, \quad \mathbf{1}^T\mathbf{W} = \mathbf{1}^T, \tag{5}$$

where $\mathbf{1}$ is the all-one vector, $z_{k,n}[t]$ converges to the average of the initial values of all nodes [12], such as $\lim_{t\to\infty} z_{k,n}[t] = \frac{1}{K}\sum_{i=1}^{K} z_{i,n}[0] = \frac{1}{K}\sum_{i=1}^{K} z_{i,n} = z_n$. As a result, all nodes can obtain the average filter output z_n, even if all nodes are not always connected to each other. Note that because nodes only communicate with their neighboring nodes, which are close to each other, they do not consume large amounts of resources to achieve average consensus.

3. Equalization algorithm

3.1 Centralized blind equalization

The kth node has one FIR filter $\mathbf{g}_k$ and its output, $z_{k,n}$ in (2), can be represented by three components, namely the desired output, ISI, and noise components, as follows:

$$z_{k,n} = \underbrace{\mathbf{g}_k{}^H \tilde{\mathbf{h}}_{k,d+1} s_{n-d}}_{\text{desired}} + \underbrace{\mathbf{g}_k{}^H \tilde{\mathbf{H}}_k \tilde{\mathbf{s}}_n}_{\text{ISI}} + \underbrace{\mathbf{g}_k{}^H \mathbf{v}_{k,n}}_{\text{noise}}, \tag{6}$$

where $\tilde{\mathbf{h}}_{k,d+1} = [\underbrace{0 \ldots 0}_{(d-L)} \ h_{k,L} \ldots \ h_{k,0} \ \underbrace{0 \ldots 0}_{(M-d-1)} \ ]^T$ is the $d+1$th column of $\mathbf{H}_k$, $\tilde{\mathbf{H}}_k$ is the remaining part of $\mathbf{H}_k$, $\tilde{\mathbf{s}}_n$ denotes a vector obtained by deleting s_{n-d} from $\mathbf{s}_n$, and d is the decision delay satisfying $M \geq d+1 \geq L+1$. Here, we consider how to update the adaptive filter $\mathbf{g}_k$ to suppress the ISI component without the knowledge of $\mathbf{H}_k$ and the use of training symbols.

To obtain the filters $\mathbf{g}_k$ blindly for the distributed equalization, we apply the MVDR principle [13] developed for centralized blind equalization. Suppose that we obtain the averaged filter output $z_n = \frac{1}{K}\mathbf{g}^H \mathbf{y}_n$ after achieving the average consensus, where $\mathbf{g} = [\mathbf{g}_1^T \ldots \mathbf{g}_K^T]^T \in C^{KM\times 1}$ and $\mathbf{y}_n = [\mathbf{y}_{1,n}^T \cdots \mathbf{y}_{K,n}^T]^T \in C^{KM\times 1}$. According to [13], the filter coefficient $\mathbf{g}$ is determined such that the filter output variance is minimized under a single constraint,

$$\min_{\mathbf{g}} \mathrm{E}[|z_n|^2], \quad \text{s.t. } \mathbf{g}^H \tilde{\mathbf{h}}_{d+1} = 1, \tag{7}$$

where $\tilde{\mathbf{h}}_{d+1} = [\tilde{\mathbf{h}}_{1,d+1}^T \ \cdots \ \tilde{\mathbf{h}}_{K,d+1}^T]^T$. The constraint in (7) comes from the distortionless response condition that guarantees that the desired component in $z_{k,n}$ is distortionless, that is, $\sum_{k=1}^K \mathbf{g}_k^H \tilde{\mathbf{h}}_{k,d+1} = 1$. The solution to (7) is given by [13]

$$\mathbf{g}_{\mathrm{SCMVDR}} = \left(\tilde{\mathbf{h}}_{d+1}^H \mathbf{R}_y^{-1} \tilde{\mathbf{h}}_{d+1}\right)^{-1} \mathbf{R}_y^{-1} \tilde{\mathbf{h}}_{d+1}, \tag{8}$$

where $\mathbf{R}_y = \mathrm{E}[\mathbf{y}_n \mathbf{y}_n^H]$. Because the channel $\tilde{\mathbf{h}}_{d+1}$ is unknown in advance, it should be estimated in a blind manner. Let us parameterize $\tilde{\mathbf{h}}_{d+1}$ as $\tilde{\mathbf{h}}_{d+1} = \bar{\mathbf{C}}\hat{\mathbf{h}}$ where

$$\bar{\mathbf{C}} = \mathrm{diag}\{\underbrace{\mathbf{C} \ldots \mathbf{C}}_{K}\}, \quad \mathbf{C} = \begin{bmatrix} \mathbf{0}_{(d-L)\times(L+1)} \\ \mathbf{J}_{(L+1)\times(L+1)} \\ \mathbf{0}_{(M-d-1)\times(L+1)} \end{bmatrix}, \quad \mathbf{J} = \begin{bmatrix} 0 & & 1 \\ & \iddots & \\ 1 & & 0 \end{bmatrix},$$

and $\hat{\mathbf{h}}$ is the estimation of the channel vector $\mathbf{h}$ defined by $\mathbf{h} = [\mathbf{h}_1^T \ \cdots \ \mathbf{h}_K^T]^T$ with $\mathbf{h}_k = [h_{k,0} \ \cdots \ h_{k,L}]^T$. The estimation $\hat{\mathbf{h}}$ that maximizes the minimum variance achieved by (8) is obtained by the eigenvector corresponding to the minimum eigenvalue of the matrix $\bar{\mathbf{C}}^H \mathbf{R}_y^{-1} \bar{\mathbf{C}}$. We refer to $\mathbf{g}_{\mathrm{SCMVDR}}$ as the single-constrained MVDR (SC-MVDR).

It is impossible to obtain the SC-MVDR in a distributed manner because the single constraint is related to all nodes. Therefore, we propose to decompose the single constraint of SC-MVDR into multiple sub-constraints suitable for distributed equalization without a fusion center, each of which corresponds to each node response. Specifically, we impose K constraints $\mathbf{g}_k^H \tilde{\mathbf{h}}_{k,d+1} = 1$ for $k = 1, \ldots, K$. The constraints guarantee that the desired component in $z_{k,n}$ in (6) is distortionless. Because $\tilde{\mathbf{h}}_{k,d+1}$ is unknown in advance, we introduce the unknown constant column vectors $\mathbf{u}_k$. We consider the following problem:

$$\min_{\mathbf{g}_1 \cdots \mathbf{g}_K} \mathrm{E}[|z_n|^2], \quad \text{s.t. } \mathbf{C}^H \mathbf{g}_k = \mathbf{u}_k, \ k = 1, 2 \ldots K. \tag{9}$$

The unknown vector $\mathbf{u}_k$ guarantees a constant response to the desired component, specifically $\mathbf{g}_k^H \tilde{\mathbf{h}}_{k,d+1} = \mathbf{g}_k^H \mathbf{C} \mathbf{h}_k = \mathbf{u}_k^H \mathbf{h}_k = \text{constant}$, where $\mathbf{h}_k = [h_{k,0} \ \cdots \ h_{k,L}]^T$. To solve it, we derive a Lagrange function of (9) as

$$J_1(\mathbf{g}) = \mathrm{E}[|z_n|^2] + \sum_{k=1}^{K} [\boldsymbol{\lambda}_k^H (\mathbf{C}^H \mathbf{g}_k - \mathbf{u}_k) + (\mathbf{g}_k^H \mathbf{C} - \mathbf{u}_k^H) \boldsymbol{\lambda}_k]. \tag{10}$$

where $\boldsymbol{\lambda}_k$ is the corresponding Lagrange multiplier. The filter minimizing $J_1(\mathbf{g})$ is given by

$$\mathbf{g}_{\text{MCMVDR}} = \mathbf{R}_y^{-1}\bar{\mathbf{C}}(\bar{\mathbf{C}}^H\mathbf{R}_y^{-1}\bar{\mathbf{C}})^{-1}\mathbf{u}. \tag{11}$$

The vector $\mathbf{u}$ is determined to maximize the minimum variance achieved by (11). The maximization can be achieved by solving the following problem.

$$\max_{\mathbf{u}} J_1(\mathbf{g}_{\text{MCMVDR}}) = \max_{\mathbf{u}} \frac{\mathbf{u}^H(\bar{\mathbf{C}}^H\mathbf{R}_y^{-1}\bar{\mathbf{C}})^{-1}\mathbf{u}}{\mathbf{u}^H\mathbf{u}}. \tag{12}$$

The solution to (12) is a normalized eigenvector corresponding to the minimum eigenvalue of $(\bar{\mathbf{C}}^H\mathbf{R}_y^{-1}\bar{\mathbf{C}})^{-1}$. We refer to $\mathbf{g}_{\text{MCMVDR}}$ as the multiple-constrained MVDR (MC-MVDR). Interestingly, the MC-MVDR filter (11) has a similar form as the filter based on the generalized sidelobe canceller principle in [13]. According to the discussion in [13, 14], the parameter vector $\mathbf{u}$ may converge to the channel vector $\mathbf{h}$ as the signal-to-noise ratio (SNR) increases. We confirm this numerically in Sect. 4.

3.2 Distributed blind adaptive equalization

It should be noted that MC-MVDR cannot be implemented in a distributed manner because it requires $\mathbf{R}_y$, depending on all the received signals. By implementing MC-MVDR adaptively, we can avoid the computation of $\mathbf{R}_y$. Based on the adaptive algorithm for SC-MVDR proposed in [14], we derive an adaptive algorithm for MC-MVDR, which works in distributed equalization without a fusion center. We apply the stochastic gradient method to minimize J_1 with respect to $\mathbf{g}_k$ and maximize J_1 with respect to $\mathbf{u}_k$. In the following, we assume that the average filter output z_n can be obtained by the average consensus where each node sends one signal $z_{k,n}[t]$ to the neighbor nodes. Then, we obtain the following updating equations:

$$\begin{aligned}
\mathbf{g}_k[n+1] &= \mathbf{g}_k[n] - \mu_g \nabla_{\mathbf{g}_k^*} J_1 \\
&= \mathbf{g}_k[n] - \mu_g \left(\frac{1}{K}\mathrm{E}[\mathbf{y}_{k,n} z_{k,n}^*] + \mathbf{C}\boldsymbol{\lambda}_k[n] \right), \qquad (13)\\
\mathbf{u}_k[n+1] &= \mathbf{u}_k[n] + \mu_u \left(\mathbf{I}_{2(L+1)} - \frac{\mathbf{u}_k[n]\mathbf{u}_k^H[n]}{\mathbf{u}_k^H[n]\mathbf{u}_k[n]} \right) \nabla_{\mathbf{u}_k^*} J_1 \\
&= \mathbf{u}_k[n] - \mu_u \left(\mathbf{I}_{2(L+1)} - \frac{\mathbf{u}_k[n]\mathbf{u}_k^H[n]}{\mathbf{u}_k^H[n]\mathbf{u}_k[n]} \right) \boldsymbol{\lambda}_k[n], \qquad (14)
\end{aligned}$$

where μ_g and μ_u are the step gains, and $\mathbf{I}$ denotes the identity matrix. In (14), we need to project $\nabla_{\mathbf{u}_k^*} J_1$ onto the space orthogonal to $\mathbf{u}_k[n]$. In (13) and (14), $\boldsymbol{\lambda}_k[n]$ is also updated. Suppose that it holds $\mathbf{C}^H\mathbf{g}_k[n+1] = \mathbf{u}_k[n]$. Then, multiplying both sides of (13) by $\mathbf{C}^H$, we obtain

$$\boldsymbol{\lambda}_k[n] = \frac{1}{\mu_g}\left(\mathbf{C}^H\mathbf{g}_k[n] - \mu_g \frac{1}{K}\mathbf{C}^H\mathrm{E}[\mathbf{y}_{k,n} z_{k,n}^*] - \mathbf{u}_k[n] \right), \tag{15}$$

because $\mathbf{C}^H\mathbf{C} = \mathbf{I}$. Substituting (15) into (13) and (14), and using an instantaneous approximation of the expectation, the updating equations become

$$\begin{aligned}
\mathbf{g}_k[n+1] &= \mathbf{g}_k[n] - \mu_g \left\{ \frac{1}{K}\mathbf{y}_{k,n} z_{k,n}^* + \mathbf{C}\left(\frac{1}{\mu_g}\left(\mathbf{C}^H\mathbf{g}_k[n] - \mu_g\frac{1}{K}\mathbf{C}^H\mathbf{y}_{k,n}z_{k,n}^* - \mathbf{u}_k[n] \right)\right)\right\} \\
&= \left(\mathbf{I}_{2M} - \mathbf{C}\mathbf{C}^H\right)\left(\mathbf{g}_k[n] - \mu_g\frac{1}{K}\mathbf{y}_{k,n}z_{k,n}^* \right) + \mathbf{C}\mathbf{u}_k[n] \qquad (16)\\
\mathbf{u}_k[n+1] &= \mathbf{u}_k[n] + \frac{\mu_u}{\mu_g}\left(\mathbf{I}_{2(L+1)} - \frac{\mathbf{u}_k[n]\mathbf{u}_k^H[n]}{\mathbf{u}_k^H[n]\mathbf{u}_k[n]} \right)\left\{ \mu_g\frac{1}{K}\mathbf{C}^H\mathbf{y}_{k,n}z_{k,n}^* - \mathbf{C}^H\mathbf{g}_k[n] \right\}. \qquad (17)
\end{aligned}$$

We normalize $\mathbf{u}_k[n]$ at each iteration as

$$\mathbf{u}_k[n+1] = \frac{\mathbf{u}_k[n+1]}{\|\mathbf{u}_k[n+1]\|}. \tag{18}$$

The proposed algorithm can be summarized as follows:

Step 1: $n = 0$, initialize $\mathbf{g}_k[0]$ and $\mathbf{u}_k[0]$, determine the step gain μ_g, μ_u, the number of iterations R for average consensus, and the number of iterations I for equalization.

Step 2: Obtain the filter output $z_{k,n}$ using (2).

Step 3: Set $z_{k,n}[0] = z_{k,n}$ and update $z_{k,n}[t+1]$ R times by using (4) to obtain $z_{k,n}[R]$ while sending $z_{k,n}[t]$ to the neighbor nodes. By making hard decision on $z_{k,n}[R]$, we obtain the data symbol estimation $\hat{s}_{n-d}$.

Step 4: Set $z_{k,n} = z_{k,n}[R]$, and update the filter coefficients according to (16), (17), and (18).

Step 5: $n = n + 1$, return to **Step 2** until n reaches I.

In the following, we refer to the proposed algorithm as the distributed MVDR (d-MVDR) algorithm.

Here, some comments are made regarding the d-MVDR algorithm. First, as can be seen in (16) and (17), each node does not require the received signals of the other nodes, unlike the centralized equalizer described in Sect. 3.1. In addition, the d-MVDR algorithm requires no training symbols to update the filters. That is, the d-MVDR algorithm can be regarded as a blind adaptive algorithm for distributed equalization. Second, each node uses only one FIR filter $\mathbf{g}_k$ and sends only one signal $z_{k,n}$ to the other nodes. This simplicity is the major advantage compared to the conventional approach. Third, we can expect superior bit error performance because multiple constraints can enhance the desired component. This is in contrast to the conventional d-MRE [11], which considers only ISI suppression, disregarding the desired component enhancement.

4. Simulation results

This section numerically confirms the validity of the proposed d-MVDR algorithm. The received SNR is defined as $\mathrm{SNR} = K\frac{\sigma_s^2 p_0}{\sigma_v^2}$, where $\sigma_s^2 = \mathrm{E}[|s_n|^2]$ is the transmitted signal power, $\sigma_v^2 = \mathrm{E}[|v_{k,n}|^2]$ is the noise variance, and $p_0 = \sum_{j=0}^{L} \mathrm{E}[|h_{k,j}|^2]$. Unless otherwise noted, we used the following simulation parameters: the modulation scheme is quadrature phase-shift keying (QPSK), the channel order $L = 2$, the filter length $M = 7$, the number of nodes $K = 5$, SNR = 16dB, the decision delay $d = 4$, the transmitted signal power $\sigma_s^2 = 1$, and we chose the step gain from $\mu_g = \mu_u \in \{0.005, 0.01, 0.02, 0.05, 0.1\}$ such that the lowest bit error rate (BER) is achieved. The BER was computed by averaging the error bits over 10^3 trials, each with 10^3 QPSK symbols. Each trial has different channels where $h_{k,j}$ were generated as complex Gaussian random variables with zero mean and variance $\sigma_h^2 = 1$.

First, we consider the BER performance of the centralized equalizers, which can be regarded as the lowest BER achieved by the distributed equalizers. Figure 2 shows the BER performances of SC-MVDR in (8), MC-MVDR in (11), and the minimum mean-squared error (MMSE) equalizer. The performance of SC-MVDR, which can be constructed blindly, is almost the same as that of MMSE, which requires either the transmission of the training symbols or a knowledge of the channels. From the comparison between SC-MVDR and MC-MVDR, it can be confirmed that there is a 2-dB loss due to the use of multiple constraints.

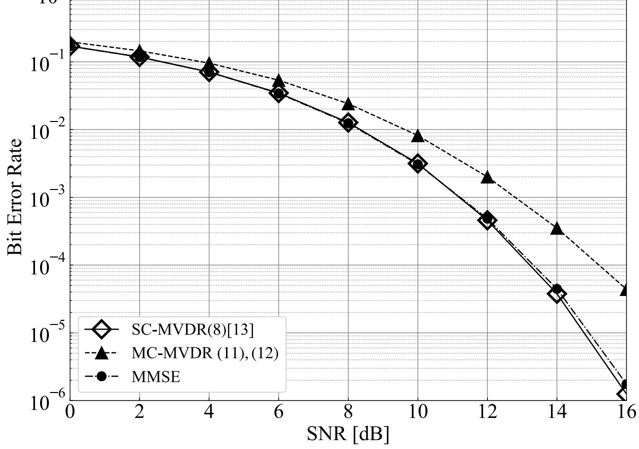

Fig. 2. BER performance comparison of centralized equalizers.

Next, we confirm whether the parameter vector $\mathbf{u}$ converges to the channel vector $\mathbf{h}$. Figure 3 shows the channel estimation performance as a function of SNR and the number of nodes K, where the channel estimation error is defined as $\epsilon = E[\|\mathbf{u} - \frac{\mathbf{h}^H\mathbf{u}}{\|\mathbf{h}\|^2}\mathbf{h}\|^2]$. It can be confirmed that the estimation error decreases as either K or SNR increases. As shown later, the channel estimation improvement results in an equalization performance improvement as either K or SNR increases.

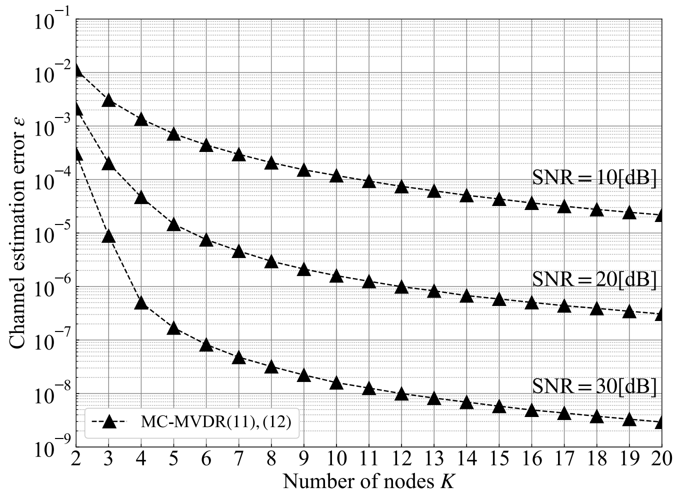

Fig. 3. Effect of the number of nodes and SNR on the channel estimation error.

In the following, we consider the performance of the proposed d-MVDR algorithm. We assume that $K = 5$ nodes are located on a line. We employ the Metropolis–Hasting weights [12] given by

$$w_{i,j} = \begin{cases} \frac{1}{1+\max\{d_i, d_j\}} & j \in \mathcal{N}_i \\ 1 - \sum_{i \neq j} w_{i,j} & j = i \\ 0 & \text{otherwise} \end{cases}$$

where d_k represents the number of nodes connected to the kth node. The convergence property of the average consensus is demonstrated in Fig. 4. We chose the number of iterations for equalization $I = 5 \times 10^3$, and the step gain $\mu_g = \mu_u = 0.02$. As the number of iterations for average consensus R increases, the BER decreases and converges after at most $R = 15$ iterations. Thus, in the following simulations, we set $R = 15$.

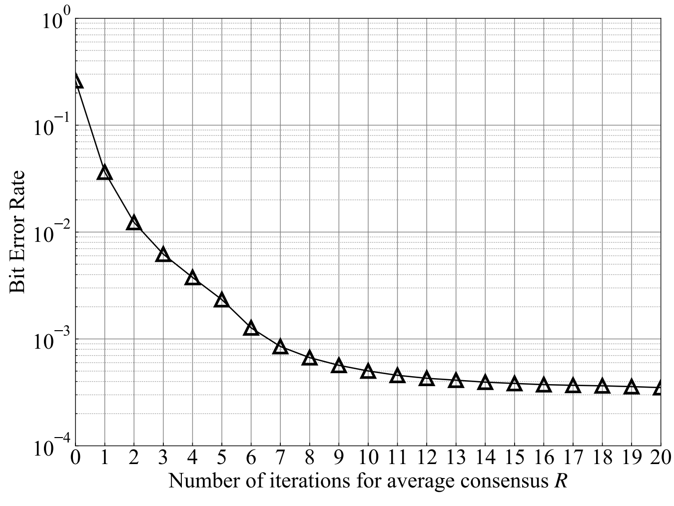

Fig. 4. Convergence property of average consensus.

Figurer 5 shows the influence of the decision delay d, where $I = 5 \times 10^3$, $\mu_g = \mu_u = 0.02$, and d is chosen such that it satisfies the condition $M \geq d + 1 \geq L + 1$. This simulation result shows that the

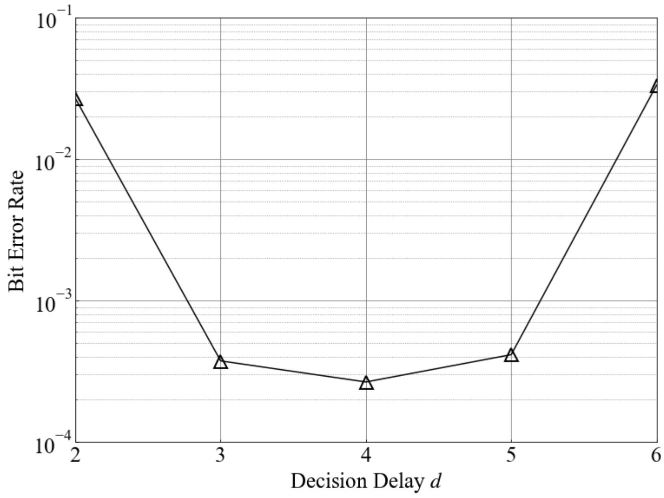

Fig. 5. Influence of decision delay d.

decision delay significantly influences the performance, and the best choice is $d = 4$, which provides the minimum bit error rate. It coincides with a heuristic choice $d = \lfloor \frac{L+M}{2} \rfloor = 4$, which was employed in the literature on channel equalization [15], where $\lfloor \cdot \rfloor$ denotes the floor function. Intuitively, the choice allows us to combine delayed waves to achieve multipath diversity efficiently. The adjustable decision delay is a major factor in the superior performance of the proposed algorithm compared to the conventional d-MRE [11], in which the delay cannot be adjusted and was forced to be $d = 0$.

Figure 6 shows the influence of the length of filter M, where the decision delay is chosen as $d = \lfloor \frac{L+M}{2} \rfloor$. The other simulation parameters are $I = 5 \times 10^3$ for $3 \leq M \leq 7$, 2×10^4 for $8 \leq M \leq 12$, and $\mu_g = \mu_u = 0.02$ for $3 \leq M \leq 7$, 0.005 for $8 \leq M \leq 12$. There is a performance gap between $M = 4$ and 5. The reason might be that frequency diversity is more pronounced for $M \geq 5$. From this figure, M should be at least 5. The optimization of the filter length M by considering the performance and complexity is worth investigating in the future.

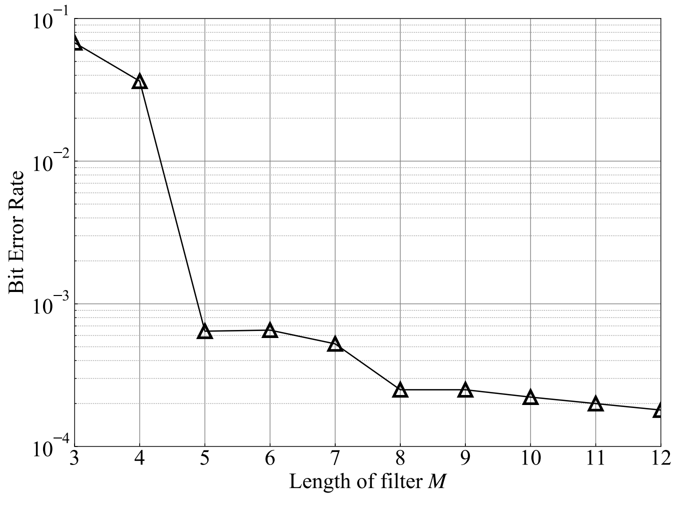

Fig. 6. Influence of the length of filter M.

Figure 7 shows the convergence property of adaptive distributed algorithms, where the proposed d-MVDR used $\mu_g = \mu_u = 0.02$, and d-MRE [11] used the step gain $\mu = 0.02$ and the delay parameter $P = 8$. We can observe that d-MVDR converges at $I = 5 \times 10^3$ iterations. However, the performance of d-MRE does not improve as the number of iterations increases.

Figure 8 shows the BER performance comparison where the simulation parameters are summarized in Table I. We can observe that the performance of the proposed d-MVDR is slightly worse than that of MC-MVDR. Because the performance gap is due to the fluctuation of $\mathbf{g}_k$ and $\mathbf{u}_k$, it could be

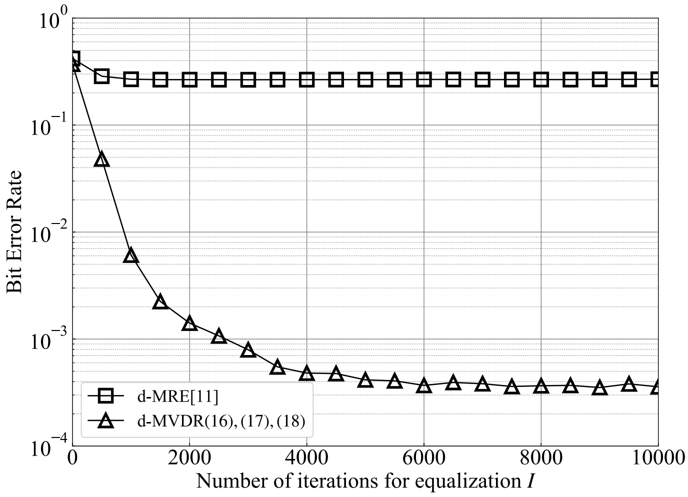

Fig. 7. Convergence property of distributed adaptive algorithms.

Table I. Simulation parameters.

Algorithm	d-MRE	d-MVDR
Number of iterations for equalizaiotn I	5×10^3	5×10^3
Step gain	$\mu = 0.02$	$\mu_g = \mu_u = 0.02$
Delay of referenced filter P	8	-
Decision delay d	0	4

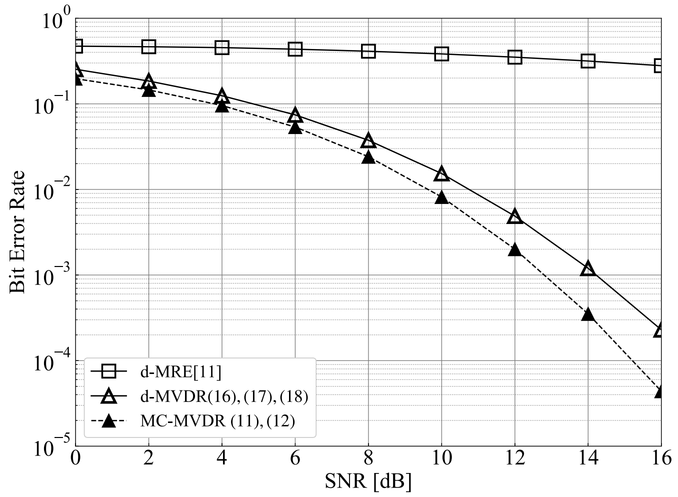

Fig. 8. BER performance comparison.

reduced by adjusting the step sizes dynamically. The figure also reveals that the proposed d-MVDR is significantly better than d-MRE.

Figure 9 shows the influence of the number of nodes K, where $R = 10$ for $K = 3$, $R = 15$ for $K = 5$, and $R = 50$ for $K = 10$; $I = 2 \times 10^3$ for $K = 3$, $I = 5 \times 10^3$ for $K = 5$, and $I = 4 \times 10^4$ for $K = 10$; $\mu_g = \mu_u = 0.05$ for $K = 3$, $\mu_g = \mu_u = 0.02$ for $K = 5$, and $\mu_g = \mu_u = 0.005$ for $K = 10$. This result is consistent with Fig. 3; the BER performance improves as K increases owing to the reduction of the channel estimation error and the increase in spatial diversity.

5. Conclusion

This study proposed a distributed equalization method based on the MVDR principle, and derived a blind adaptive algorithm. The complexity of the proposed algorithm is simpler than that of a conventional algorithm, as each node requires only one FIR filter and sends only one signal. The simulation results revealed that the performance of the proposed algorithm is significantly improved

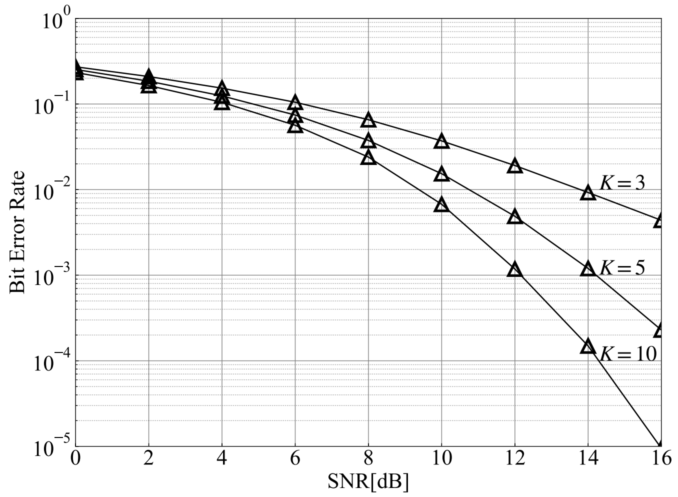

Fig. 9. Influence of the number of nodes K.

compared with that of a conventional algorithm.

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