

Neural network-based time-domain equalization without training signal in OFDM systems without CP

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Abstract: This paper proposes a neural network-based time-domain equalizer (TEQ) in OFDM systems. The proposed TEQ based on the minimum output energy criterion does not require the transmission of a training signal or the insertion of a cyclic prefix to suppress inter-symbol interference; thus, the proposed TEQ does not degrade bandwidth efficiency. Further, an arbitrary decision delay and multiple receive antennas are introduced to improve the bit error rate performance. By simulation, we show that the proposed TEQ is significantly superior to a conventional scheme.

Keywords: OFDM, time-domain equalization, neural network

Classification: Wireless Communication Technologies

References

- [1] M. Toeltsch and A.F. Molisch, "Efficient OFDM transmission without cyclic prefix over frequency-selective channels," Proc. IEEE PIMRC, pp. 1363–1367, London, Sept. 2000. DOI: [10.1109/PIMRC.2000.881641](https://doi.org/10.1109/PIMRC.2000.881641)
- [2] T. O'Shea and J. Hoydis, "An introduction to deep learning for the physical layer," *IEEE Trans. Cogn. Commun. Netw.*, vol. 3, no. 4, pp. 563–575, Oct. 2017. DOI: [10.1109/TCCN.2017.2758370](https://doi.org/10.1109/TCCN.2017.2758370)
- [3] G.J. Gibson, S. Siu, and C.F.N. Cowan, "The application of nonlinear structures to the reconstruction of binary signals," *IEEE Trans. Signal Process.*, vol. 39, pp. 1877–1884, Aug. 1991. DOI: [10.1109/78.91157](https://doi.org/10.1109/78.91157)
- [4] H. Ye, G.Y. Li, and B.H. Juang, "Power of deep learning for channel estimation and signal detection in OFDM system," *IEEE Wireless Commun. Lett.*, vol. 7, no. 1, pp. 114–117, Feb. 2018. DOI: [10.1109/LWC.2017.2757490](https://doi.org/10.1109/LWC.2017.2757490)
- [5] J. Liu, K. Mei, X. Zhang, D. Ma, and J. Wei, "Online extreme learning machine-based channel estimation and equalization for OFDM systems," *IEEE Commun. Lett.*, vol. 23, no. 7, pp. 1276–1279, July 2019. DOI: [10.1109/LCOMM.2019.2916797](https://doi.org/10.1109/LCOMM.2019.2916797)
- [6] R. Koizumi, T. Miyajima, and K. Yamanaka, "Constrained blind equalizer using adaptive nonlinear filter," *Electron. and Commun. in Japan*, pt. 3, vol. 87, no. 3, pp. 17–29, March 2004. DOI: <https://doi.org/10.1002/ecjc.10145>
- [7] G. Xu, H. Liu, L. Tong, and T. Kailath, "A least-squares approach to blind channel identification," *IEEE Trans. Signal Process.*, vol. 43, pp. 2982–2993, Dec. 1995.

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1 Introduction

Orthogonal frequency division multiplexing (OFDM) has been adopted widely in various wireless standards; however, it reduces bandwidth efficiency owing to the insertion of a cyclic prefix (CP) to suppress inter-symbol interference (ISI) in frequency-selective channels. Accordingly, OFDM systems without CP have been studied to improve bandwidth efficiency, where ISI suppression is indispensable [1].

Meanwhile, recently, applications of machine learning (ML) to signal detection in wireless communications have been considered [2]. In [3], it was shown that the nonlinearity of the neural network (NN) is needed to achieve optimum detection, which is impossible to implement via conventional linear methods. The approaches employed in ML-based detectors are divided into two types: a pre-training approach and an online training approach.

In [4], an OFDM signal detector using a deep neural network (DNN) has been presented as a pre-training approach, where the NN is learned by using training samples generated from a number of channel models. The DNN-based detector can be superior to linear detectors even when impairments such as nonlinear distortion exist. However, the detection performance is degraded owing to the mismatch between real environments and channel models used for pre-training.

In [5], an OFDM signal detector based on an extreme learning machine (ELM) has been proposed as an online training approach, where the ELM is learned by received signals obtained through a current channel. The ELM-based detector demonstrates performance superior to that of the DNN-based and conventional linear detectors. Moreover, in single-carrier systems, a time-domain channel equalizer (TEQ) using a NN has been proposed as another online training approach [3]. The NN-TEQ is superior to a conventional linear TEQ. However, these online training schemes require the transmission of large numbers of known training signals, which cause significant bandwidth efficiency degradation. To overcome this disadvantage, a blind NN-TEQ requiring no known training signals for online training has been considered in [6]. The blind NN-TEQ demonstrates performance equivalent to that of the nonblind NN-TEQ in [3]; however, significant performance degradation occurs when the first path gain of the channel impulse response (CIR) is small.

In this paper, through modification of the blind NN-TEQ in [6], we propose a novel NN-TEQ without known training signals in OFDM systems without CP. The basic idea of the proposed TEQ is to suppress ISI by minimizing the equalizer output energy while maintaining the desired signal component. To prevent performance degradation owing to a small first path gain, we introduce an arbitrary decision delay and multiple receive antennas. Unlike [3, 4, 5], the proposed TEQ requires no transmission of known training signals for training.

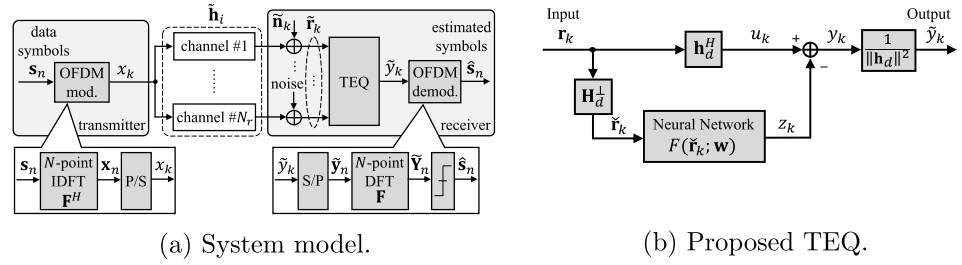


Fig. 1. Structure of OFDM system without CP and proposed TEQ.

2 System model

Figure 1(a) depicts the system model using a transmitter with a single antenna and a receiver with N_r antennas. We consider an OFDM system with N subcarriers. The complex-valued data symbols s_k with variance $\sigma_s^2 = \mathbb{E}[|s_k|^2]$ are grouped into a data block $\mathbf{s}_n = [s_{nN} \ s_{nN+1} \ \cdots \ s_{nN-(N-1)}]^T \in \mathbb{C}^N$. Subsequently, the time-domain signal of the n th block obtained via the N -point inverse discrete Fourier transform (IDFT) of \mathbf{s}_n is transmitted; $\mathbf{x}_n = \mathbf{F}^H \mathbf{s}_n = [x_{nN} \ x_{nN+1} \ \cdots \ x_{nN-(N-1)}]^T$, where \mathbf{F} denotes the N -point DFT matrix whose (l, m) th element is $\exp(-j2\pi lm/N)/\sqrt{N}$. We assume that the desired signal at a receiver is x_{k-d} at time k , where d is an arbitrary decision delay that takes an integer value within $[0, L_w + L_h - 1]$. At the transmitter, the transmitted signal x_k is transmitted without the insertion of a CP.

After x_k passes through frequency-selective channels, the time-domain received signal suffered from ISI at all receive antennas is observed as

$$\tilde{\mathbf{r}}_k = [r_{1,k} \ \cdots \ r_{N_r,k}]^T = \sum_{i=0}^{L_h} \tilde{\mathbf{h}}_i x_{k-i} + \tilde{\mathbf{n}}_k \in \mathbb{C}^{N_r}, \quad (1)$$

where L_h is the order of the CIR, $r_{j,k} = \sum_{i=0}^{L_h} h_{j,i} x_{k-i} + n_{j,k}$ is the received signal at the $j \in \{1, \dots, N_r\}$ th receive antenna, $\tilde{\mathbf{h}}_i = [h_{1,i} \ \cdots \ h_{N_r,i}]^T \in \mathbb{C}^{N_r}$, $\tilde{\mathbf{n}}_k = [n_{1,k} \ \cdots \ n_{N_r,k}]^T \in \mathbb{C}^{N_r}$, $h_{j,i}$ is the i th path gain of the CIR between the transmit antenna and the j th receive antenna, $n_{j,k} \in \mathcal{CN}(0, \sigma_n^2)$, $\sigma_n^2 = \mathbb{E}[|n_{j,k}|^2]$ is the additive white Gaussian noise at the j th receive antenna. A signal that collected L_w successive samples is input to a TEQ as

$$\mathbf{r}_k = [\tilde{\mathbf{r}}_k^T \ \cdots \ \tilde{\mathbf{r}}_{k-(L_w-1)}^T]^T = \mathbf{h}_d x_{k-d} + \check{\mathbf{H}} \check{\mathbf{x}}_k + \mathbf{n}_k \in \mathbb{C}^{N_r L_w}, \quad (2)$$

where $\check{\mathbf{x}}_k$ is the vector removed x_{k-d} from $\mathbf{x}_k \triangleq [x_k \ \cdots \ x_{k-d} \ \cdots \ x_{k-(L_w+L_h-1)}]^T \in \mathbb{C}^{L_w+L_h}$, $\check{\mathbf{H}} = [\tilde{\mathbf{h}}_0 \ \cdots \ \tilde{\mathbf{h}}_{L_h}]$, $\mathbf{H} \triangleq \text{toeplitz}(\check{\mathbf{H}}, L_w) = [\mathbf{h}_0 \ \cdots \ \mathbf{h}_d \ \cdots \ \mathbf{h}_{L_w+L_h-1}] \in \mathbb{C}^{N_r L_w \times (L_w+L_h)}$, \mathbf{h}_d is the $(d+1)$ th column vector of \mathbf{H} , $\check{\mathbf{H}}$ is the matrix removed \mathbf{h}_d from \mathbf{H} , and $\mathbf{n}_k \triangleq [\tilde{\mathbf{n}}_k^T \ \cdots \ \tilde{\mathbf{n}}_{k-(L_w-1)}^T]^T$. The signal \mathbf{r}_k contains the desired signal, ISI, and noise components. The purpose of the TEQ is to detect x_{k-d} from \mathbf{r}_k by suppressing ISI.

Subsequently, the output signal from the TEQ is obtained as \tilde{y}_k . After \tilde{y}_k are grouped into a block $\tilde{\mathbf{y}}_n = [\tilde{y}_{nN} \ \tilde{y}_{nN+1} \ \cdots \ \tilde{y}_{nN-(N-1)}]^T \in \mathbb{C}^N$, a frequency-domain block $\tilde{\mathbf{Y}}_n = \mathbf{F} \tilde{\mathbf{y}}_n \in \mathbb{C}^N$ is obtained by taking the N -point DFT of $\tilde{\mathbf{y}}_n$. Finally, the estimated symbol $\hat{\mathbf{s}}_n = [\hat{s}_{nN} \ \hat{s}_{nN+1} \ \cdots \ \hat{s}_{nN-(N-1)}]^T \in \mathbb{C}^N$ is obtained by the hard-decision of $\tilde{\mathbf{Y}}_n$.

3 Proposed TEQ

Figure 1(b) shows the structure of the proposed TEQ using a NN. The proposed TEQ is constructed in two parts: parts of the maximum ratio combining (MRC) and an ISI replica generator.

In the upper branch, called the part of the MRC for the desired signal component, the input signal to the TEQ \mathbf{r}_k is processed as

$$u_k = \mathbf{h}_d^H \mathbf{r}_k = \|\mathbf{h}_d\|^2 x_{k-d} + \mathbf{h}_d^H \check{\mathbf{H}} \check{\mathbf{x}}_k + \mathbf{h}_d^H \mathbf{n}_k. \quad (3)$$

In (3), by setting $d \neq 0$ appropriately, the desired signal component can be enhanced compared to that in the NN-TEQ [6]. It can also be enhanced further by using multiple receive antennas. The introduction of the decision delay and multiple antennas avoids the performance degradation owing to a small first path gain. Note that setting $d = 0$ and $N_r = 1$ is equivalent to the case in the NN-TEQ in [6]. In the lower branch, called the ISI replica generator, first, the desired signal component is removed from \mathbf{r}_k :

$$\hat{\mathbf{r}}_k = \mathbf{H}_d^\perp \mathbf{r}_k = \mathbf{H}_d^\perp \check{\mathbf{H}} \check{\mathbf{x}}_k + \mathbf{H}_d^\perp \mathbf{n}_k \in \mathbb{C}^{N_r L_w}, \quad (4)$$

where $\mathbf{H}_d^\perp \triangleq \mathbf{I}_{N_r L_w} - (\mathbf{h}_d \mathbf{h}_d^H / \|\mathbf{h}_d\|^2) \in \mathbb{C}^{N_r L_w}$ is orthogonal to \mathbf{h}_d , \mathbf{I}_M is an $M \times M$ identity matrix. Then, the input to a NN $\hat{\mathbf{r}}_k$ contains only the ISI and noise components. Actually, the input to a NN is a real-valued vector $\check{\mathbf{r}}_k \in \mathbb{R}^{2N_r L_w}$ consisting of the real and imaginary parts of the complex-valued vector $\hat{\mathbf{r}}_k$. Then, the NN computes its output which is given by $\mathbf{z}_k \triangleq F(\check{\mathbf{r}}_k; \mathbf{w}) = [z_k^{\text{Re}} \ z_k^{\text{Im}}]^T \in \mathbb{R}^2$, where $F(\cdot)$ represents the function of a whole NN, and \mathbf{w} represents all the adjustable parameters of the NN. The output signal of the ISI replica generator is obtained by transforming from \mathbf{z}_k to the complex-valued scalar $z_k = z_k^{\text{Re}} + j z_k^{\text{Im}}$. Finally, after the difference signal of two parts is computed as

$$y_k = u_k - z_k, \quad (5)$$

the output signal from the TEQ is given by $\tilde{y}_k = y_k / \|\mathbf{h}_d\|^2$.

Let us consider a simple strategy to train the NN. In the proposed TEQ, u_k contains the desired signal, ISI, and noise components, and z_k contains only ISI and noise components. Because the desired signal component in y_k is never affected by the NN, only the ISI and noise components can be suppressed by minimizing $\mathbb{E}[|y_k|^2]$, while the desired signal component is kept unchanged. To train \mathbf{w} adaptively, instead of the mean energy $\mathbb{E}[|y_k|^2]$, we evaluate instantaneous energy $J(\mathbf{w}) \triangleq |y_k|^2 = |\text{Re}[y_k]|^2 + |\text{Im}[y_k]|^2$.

The online training process is summarized as follows: **1)** Estimate the current channel \mathbf{H} and calculate \mathbf{H}_d^\perp , **2)** Initialize \mathbf{w} randomly, **3)** Obtain y_k from \mathbf{r}_k at time k , **4)** Update \mathbf{w} by minimizing $J(\mathbf{w})$, **5)** Repeat steps **3)** to **4)** for B OFDM blocks.

Here, some comments are made regarding the proposed TEQ. First, any optimizer such as the stochastic gradient descent and Adam can be applied to minimize $J(\mathbf{w})$. Second, the current channel can be estimated by blind channel estimation (CE) methods such as in [7]. Third, the proposed TEQ needs to retrain for the new channel if the channel changes.

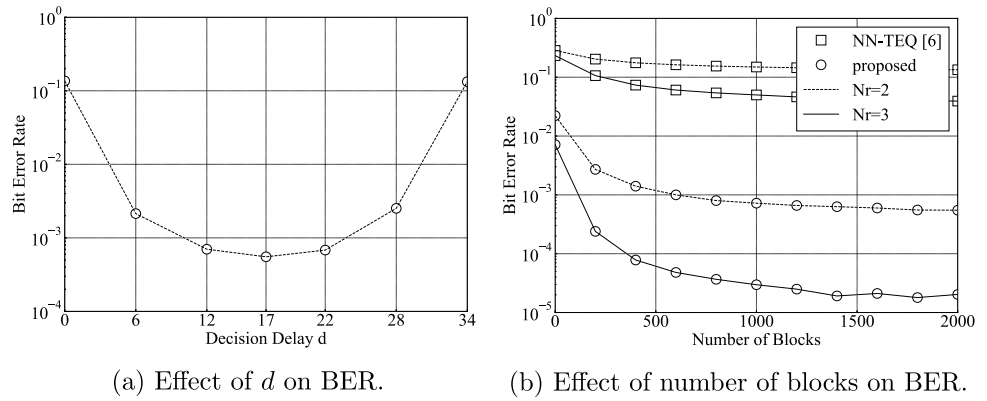


Fig. 2. Performance evaluation for proposed TEQ.

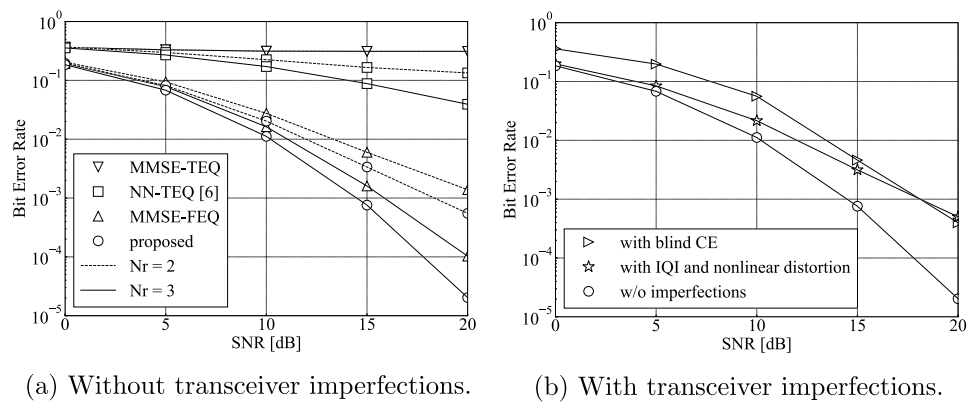


Fig. 3. BER performance for various SNR values.

4 Simulation results

We evaluate the bit error rate (BER) performance of the proposed TEQ by simulations. The BER is evaluated by the test blocks, which are different from the blocks used for online training. The average BER is obtained from 100 trials, where each trial has an independent channel realization and 10^3 test blocks. The simulation parameters were as follows: the modulation scheme was quadrature phase-shift keying (QPSK), $N = 64$, $\sigma_s^2 = 1$, $L_h = 8$, $h_{j,i} \in CN(0, \sigma_i^2)$, $\sigma_i^2 = \lambda \exp(-\alpha i)$, $i = 0, \dots, L_h$, where $\lambda = \sigma_h^2 / \sum_{i=0}^{L_h} \exp(-\alpha i)$, $\alpha = 0.1$, $\sigma_h^2 = 1$, $L_w = 27$, and the received SNR was defined as $\text{SNR} = N_r \sigma_x^2 \sigma_h^2 / \sigma_n^2$ where $\sigma_x^2 = \mathbb{E}[|x_k|^2]$. We used the NN with three fully connected layers, where the number of neurons in each layer is $2N_r L_w$, $4N_r L_w$, and 2, respectively, and the activation function in the hidden layer is $f(x) = \max(0, x)$. The optimizer was the stochastic gradient descent with momentum, and the learning rate was 5×10^{-5} , the momentum parameter was 0.8, and $B = 2000$. These parameters were determined based on the results of preliminary simulations. In Fig. 2 and Fig. 3(a), we assumed that the CIR is perfectly known at the receiver.

Figure 2(a) depicts the BER performances after training against various decision delays d , where $N_r = 2$, and SNR = 20 dB. Note that $d = 0$ corresponds to the NN-TEQ [6]. As can be seen, the BER is minimized around $d = 17$ because we benefit from the multipath diversity when \mathbf{h}_d contains many non-zero elements; thus, to achieve better performance, we set $d = \lfloor (L_w + L_h) / 2 \rfloor$ in the simulations

below. Figure 2(b) depicts the BER performance against the number of blocks used for training, where $\text{SNR} = 20$ dB. In Fig. 2(b), although the NN-TEQ [6] cannot decrease the BER even if a large number of blocks and receive antennas is used, the proposed TEQ can obtain the better BER performance as the number of blocks increases.

Figure 3(a) depicts the BER performance for various values of SNR. We compared the proposed TEQ to the NN-TEQ [6], the linear MMSE-TEQ, and the linear MMSE-frequency-domain equalizer (FEQ). As can be seen, the proposed TEQ is significantly superior to the MMSE-TEQ and NN-TEQ [6], and it also outperforms the MMSE-FEQ. Figure 3(b) depicts the BER performance of the proposed TEQ with transceiver imperfections; IQ imbalance (IQI), nonlinear distortion owing to a power amplifier (PA), and channel estimation error, where $N_r = 3$. We presented the transmitted signal with IQI as $x_{\text{IQI},k} = \alpha x_k + \beta x_k^*$, where $\alpha = \cos\phi + j\gamma\sin\phi$, $\beta = \gamma\cos\phi - j\sin\phi$, γ is the amplitude error, and ϕ is the phase error. We used the Rapp model to express PA nonlinearity, and the output of PA is presented by $x_{\text{PA},k} = x_k(1 + (|x_k|/A_0)^{2\rho})^{-1/2\rho}$, where ρ is a nonlinearity parameter, A_0 is the saturation level defined by input backoff (IBO) as $\text{IBO} = 10\log_{10}(A_0^2/\sigma_x^2)$ [dB]. The blind CE method in [7] was used, where $B_{\text{est}} = 20$ OFDM blocks were used for estimation. The simulation parameters were as follows: $\gamma = 0.2$, $\phi = 4^\circ$, $\rho = 2$, and $\text{IBO} = 3$ dB. In Fig. 3, the performance of the proposed TEQ using blind CE is degraded. Since channel estimation accuracy depends on B_{est} , the BER performance can be improved by increasing B_{est} . In addition, when IQI and nonlinear distortion exist, the BER performance is degraded in the high SNR region. To improve the performance, we should tune the hyper-parameters of the NN more carefully.

5 Conclusion

In this paper, we proposed a novel NN-based TEQ in OFDM systems without CP. The proposed TEQ does not require the transmission of training signals or the insertion of CPs. By minimizing the output energy, the proposed TEQ suppresses only the ISI and noise components, while the desired signal component remains unchanged. Simulation results show that the proposed TEQ is much superior to the conventional scheme [6] by setting an appropriate decision delay and using multiple receive antennas.